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Multiphysics Software for the Modeling of Hypersonic Flows

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USim: Advanced Fluid, Plasma and Electromagnetic Modeling on Unstructured Meshes

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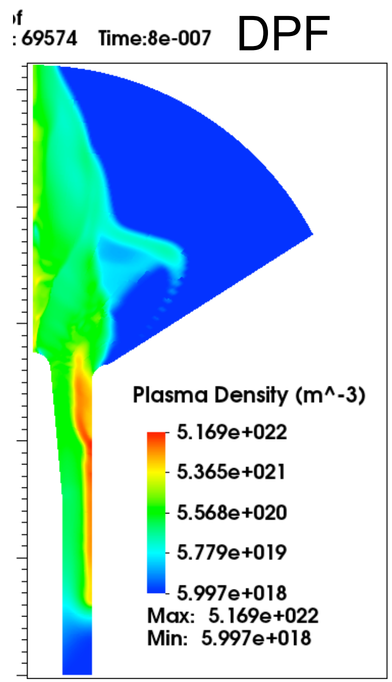
USim is a Generalized fluid plasma modeling framework developed to model the dynamics of neutral, partially-ionized and fully-ionized fluids on unstructured meshes.

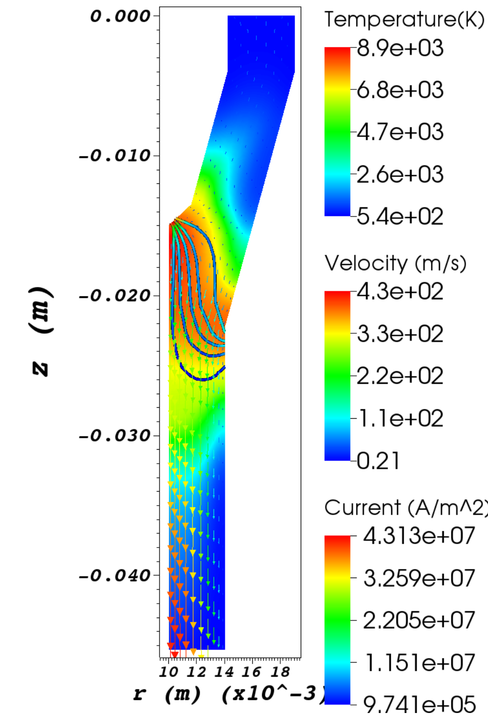
Fluid models such as Hall MHD, two-fluid plasma, and Navier-Stokes enabling increasingly detailed models of hypersonic flows and improved designs for high energy density laboratory plasma experiments

Areas of Application Include:
Hypersonic Flight
Radar Cross Section
Blackout of Reentry Vehicles
Dense Plasma Focus
Plasma Jets Modelling
Ion Sources
Magnetic Reconnection
Plasma Torches
Scram Jets

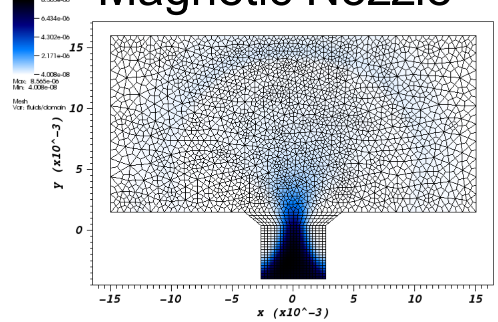
DPF



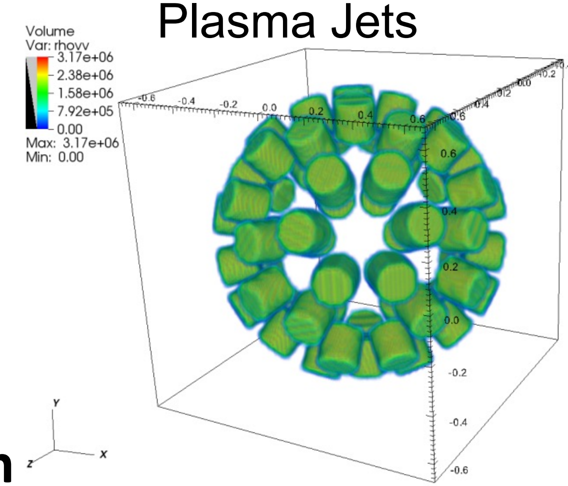
Plasma Torch



Magnetic Nozzle



Plasma Jets



www.txcorp.com/usim

USim: Equation Systems

- Multi-Fluid Equations
  - Models inviscid, viscous, and reactive flow descriptions. Several fluids can be coupled together using their collisional momentum and energy transfer. The fluids can be neutrals and/or ions.
- Single-Fluid Multi-Species System
  - Models inviscid, viscous, reactions. Single bulk fluid with multiple species transported using the bulk velocity.
- Maxwell's Equations
  - Electric and Magnetic field solver for a full description of plasma (two-fluid).
  - EM wave propagation.
- MHD Equations
  - Ideal MHD along with the generalized Ohm's law to include the hall, resitstive, and diamagnetic drift terms.
- Poisson Equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

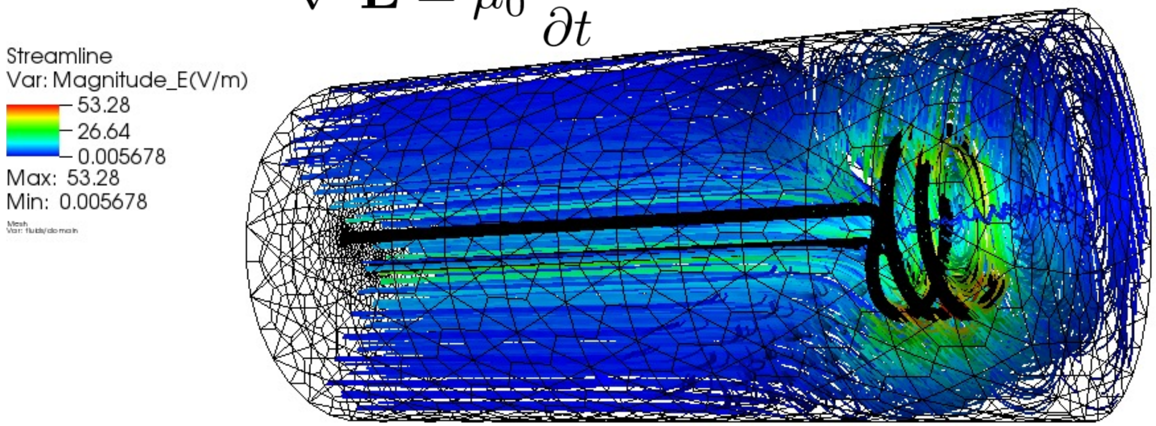
$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u}\mathbf{u} + pI + P_B) = 0$$

$$\frac{\partial e_{tot}}{\partial t} + \nabla \cdot \left((e_{tot} + p)\, \mathbf{u} + P_B . \mathbf{u}\right) = \nabla \cdot \left(\frac{1}{\sigma \mu_0^2} \mathbf{B} \times \nabla \times \mathbf{B} \right)$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \left(-\mathbf{u} \times \mathbf{B} + \frac{J}{\sigma \mu_0} \right) = 0$$

$$\frac{\partial s_e}{\partial t} + \nabla \cdot (s_e \mathbf{u}) = \frac{(\gamma - 1)}{n_e^{\gamma - 1}} \left(\frac{\mathbf{J}^2}{\sigma} \right)$$

$$\nabla^2 \mathbf{E} = \mu_0 \frac{\partial \mathbf{J}}{\partial t}$$



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USim Benefits

- USim provides scientists & engineers:
  - Ability to perform coupled fluid-plasma-electromagnetic simulations with chemistry and ablation physics
  - Simulation models for high density plasmas where kinetic simulations are not practical
- USim is built with software engineering best practices:
  - Test driven development & simulation validation
  - Object-orientated design that enables multi physics simulation
  - Cross-platform simulation engine & GUI
  - Examples & documentation for a wide range of problems

Multi-fluid Model

Mass $$\frac{\partial \rho_\alpha}{\partial t} + \nabla \cdot (\rho_\alpha \vec{u}_\alpha) = 0$$

Momentum $$\frac{\partial (\rho_\alpha \vec{u}_\alpha)}{\partial t} + \nabla \cdot (\rho_\alpha \vec{u}_\alpha \vec{u}_\alpha + p_\alpha I) = \frac{\rho_\alpha}{m_\alpha} q_\alpha \left(\vec{E} + \vec{u}_\alpha \times \vec{B} \right) + \nabla \cdot \tau_\alpha + \vec{R}_\alpha$$

Energy $$\frac{\partial (e_\alpha)}{\partial t} + \nabla \cdot (\vec{u}_\alpha (e + p_\alpha)) = \frac{\rho_\alpha}{m_\alpha} q_\alpha \vec{u}_\alpha \cdot \vec{E} + \tau_\alpha : \nabla \vec{u}_\alpha + \nabla \cdot (k_\alpha \nabla T_\alpha) + \vec{V} \cdot \vec{R}_\alpha + Q_\alpha$$

Bulk velocity $$\vec{V} = \left(\sum_i \rho_i \vec{u}_i \right) / \sum_i \rho_i$$

Momentum exchange $$\vec{R}_\alpha = -\sum_i \frac{\rho_\alpha}{m_\alpha} \mu_{\alpha i} \zeta_{\alpha i}^{-1} (\vec{u}_\alpha - \vec{u}_i)$$

Internal energy exchange $$Q_\alpha = -\sum_i 3k_B \frac{\rho_\alpha}{m_\alpha} \left[\mu_{\alpha i} / (m_\alpha + m_i) \right] \zeta_{\alpha i}^{-1} (T_\alpha - T_i)$$

Ampere's law $$\frac{\partial \vec{E}}{\partial t} - c^2 \nabla \times \vec{B} = -\frac{1}{\epsilon_0} \sum_\alpha \frac{q_\alpha \rho_\alpha \vec{u}_\alpha}{m_\alpha}$$

Faraday's law $$\frac{\partial \vec{B}}{\partial t} + \nabla \times \vec{E} = 0$$

Divergence equations $$\nabla \cdot \vec{E} = \frac{1}{\epsilon_0} \sum_\alpha q_\alpha \rho_\alpha$$

Divergence cleaning is applied to obtain divergence free fields.

Munz et.al, Computer Physics Communications 130.1 (2000): 83-117

Reentry Vehicle Analysis

Atmospheric flights of hypersonic vehicles encounter plasma environment due to shock heating.

Flow characterization is important for various design aspects including the aerothermal design, signature prediction, and blackout analysis.

Plasma interrupts communication waves, when the plasma frequency exceeds the wave frequency.

Mitigation ideas are electrophilic fluid injection, ablators, magnetic window.

USim Models for Hypersonics

- Available models & capabilities
  - 7 or 11 species air chemistry
  - Translational-vibrational energies
  - Variable thermophysical properties computed self-consistently using kinetic theory
  - Full Maxwell's equations solver
  - Parallel processing
  - Structured and unstructured grids
- Gas radiation
  - Any available empirical data can be added without any difficulty!
  - Emissivities can be computed for atomic species using PROPACEOS software.

Single Fluid Multi-Species Model

Mass $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

Momentum $$\frac{\partial (\rho \vec{u})}{\partial t} + \nabla \cdot (\rho \vec{u}\vec{u} + pI) = \nabla \cdot \tau$$

Energy $$\frac{\partial (e)}{\partial t} + \nabla \cdot (\vec{u}(e+p)) = \tau : \nabla \vec{u} + \nabla \cdot (k \nabla T)$$

$$e = \rho c_v T + \frac{1}{2}\rho \vec{u}\cdot\vec{u} + \sum_i n_i H_i$$

Equation of State (ideal gas laws)

$$\tau = -\frac{2}{3}\mu (\nabla \cdot \vec{u}) I + \mu \left(\nabla \vec{u} + (\nabla \vec{u})^T\right)$$

Species conservation $$\frac{\partial n_i}{\partial t} + \nabla \cdot (\vec{u} n_i) = s_i$$

Dynamic viscosity $$\mu_i = \frac{5}{16}\frac{\sqrt{\pi m_i k_B T}}{(\pi \sigma^2 \Omega)}$$

Thermal conductivity $$k_i = \frac{5}{2} c_{vi} \mu_i$$

Specific heat $$c_{vi} = \frac{f}{2} R_i$$

Energy density e is the sum of the internal energy, kinetic energy, and chemical energy of the fluid.

S_i is the rate of change of species due to reactions.

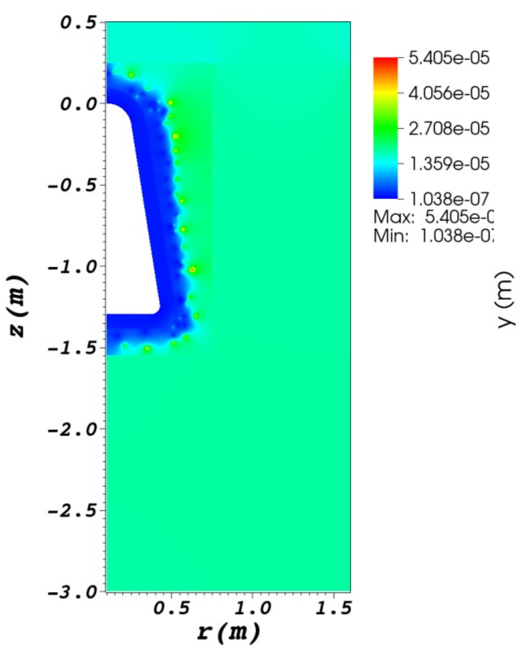
f is the number of degrees of freedom

Species will be transported with the bulk velocity u

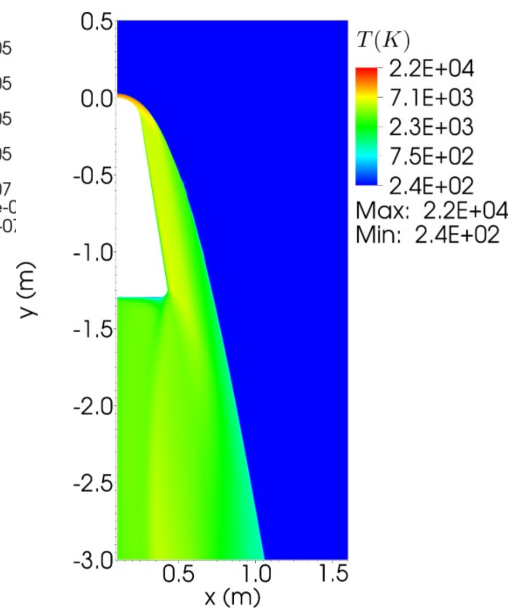
Multi-Species Simulation on RAMC

Altitude = 61 km
Mach 24
Species: N2, N, O2, O, NO, NO+, e-

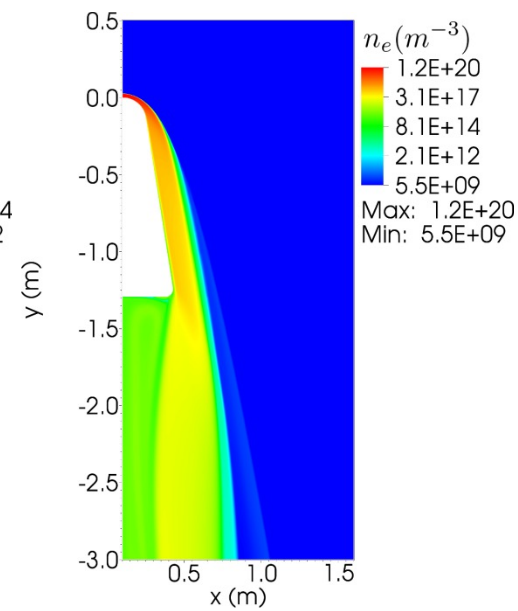
Grid area (m^2)

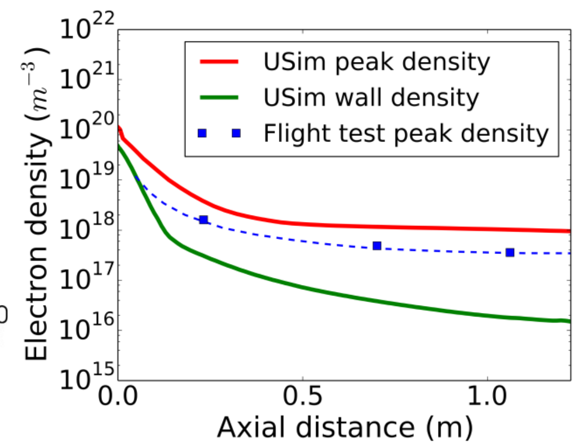


Temperature (K)



Electron density (m^{-3})





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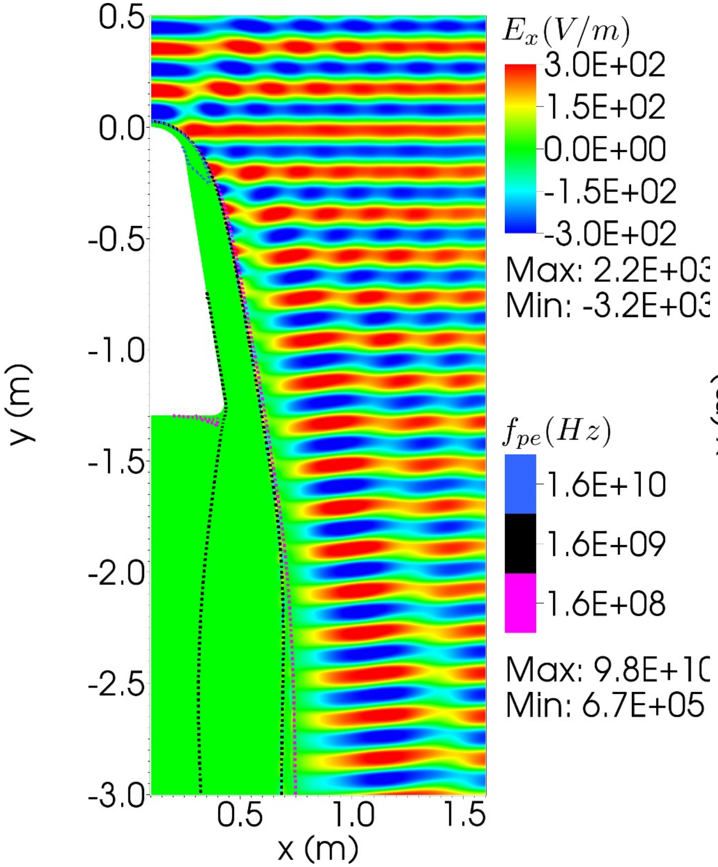
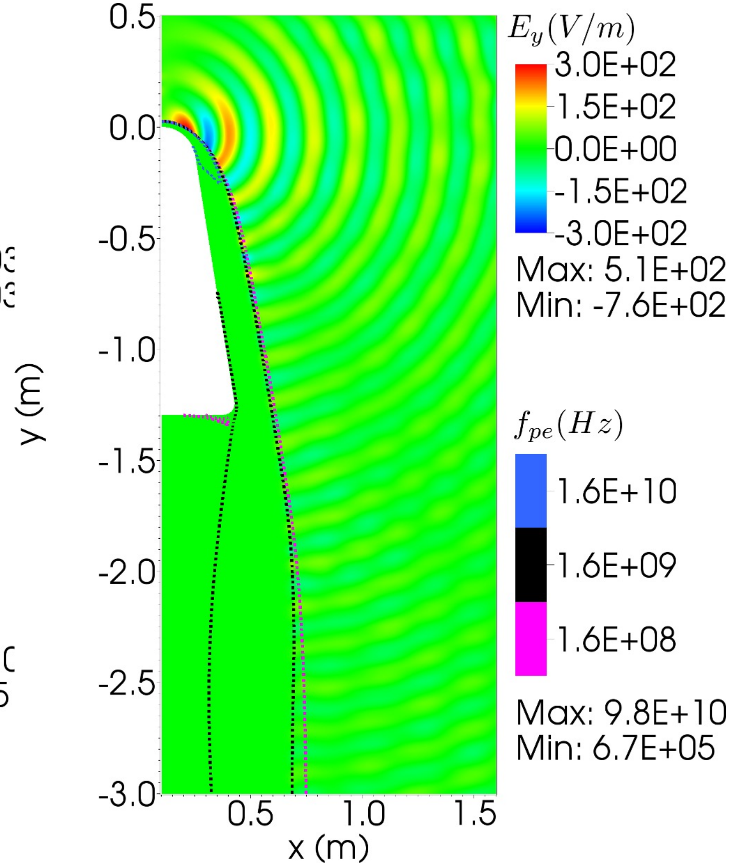
Kundrapu et.al, JSR 2015

Wave Propagation on RAMC

f = 1.6 GHz; $E_x = 300\sin(\omega t)$; plasma density > 10^{19} m^{-3}





EM wave was fully reflected by the plasma layer.

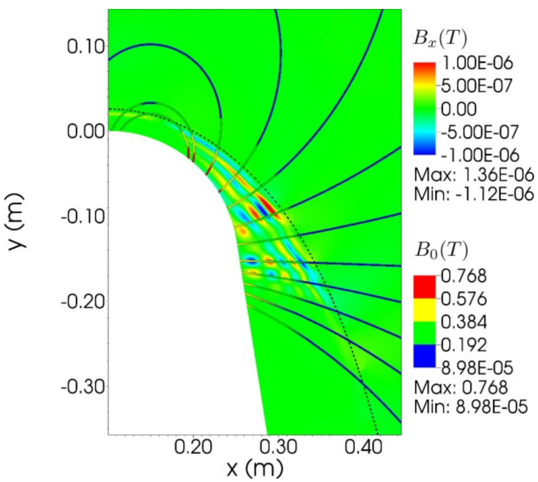
Electrophilic fluid injection

Magnetic window

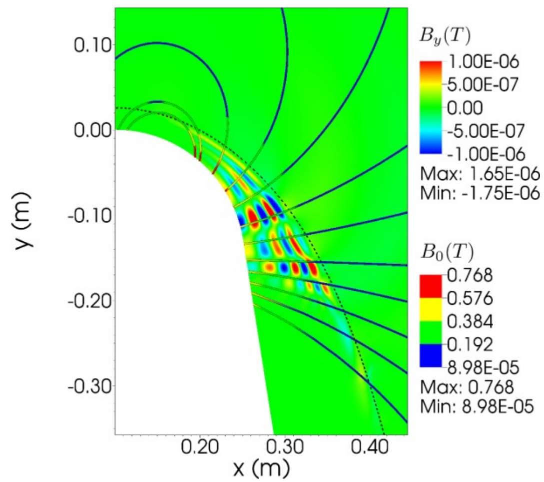
Magnetic Window on RAM C

A magnetic field of 0.7T was applied near the surface!
Wave propagates on to the surface

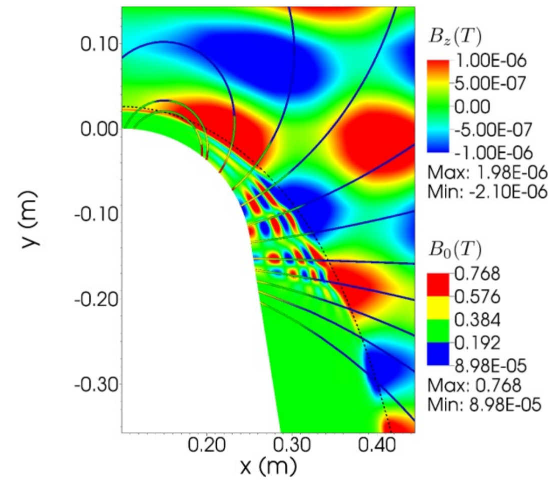
Bx (T)



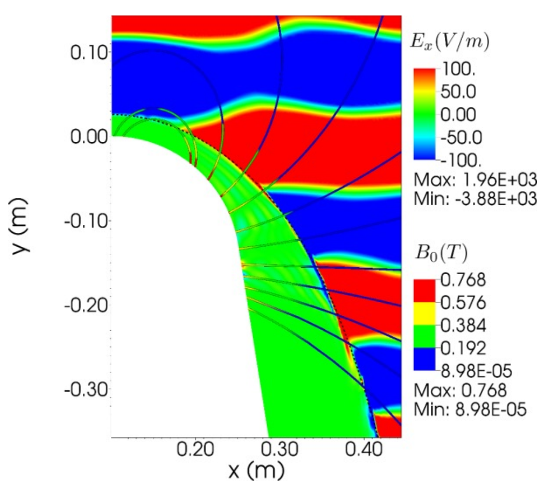
By (T)



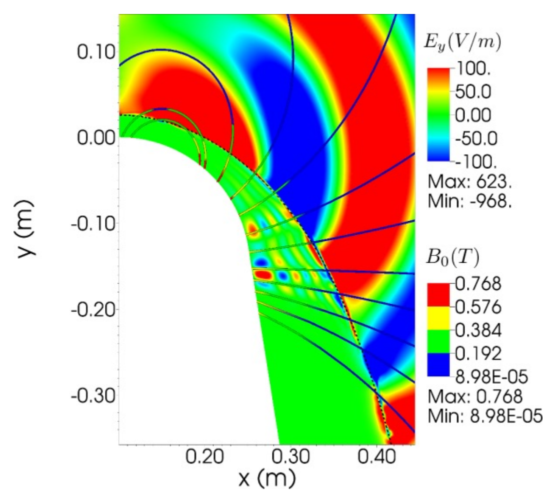
Bz (T)



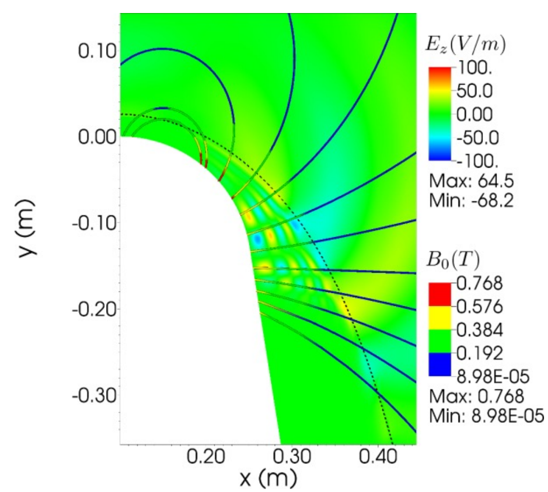
Ex (V/m)



Ey (V/m)

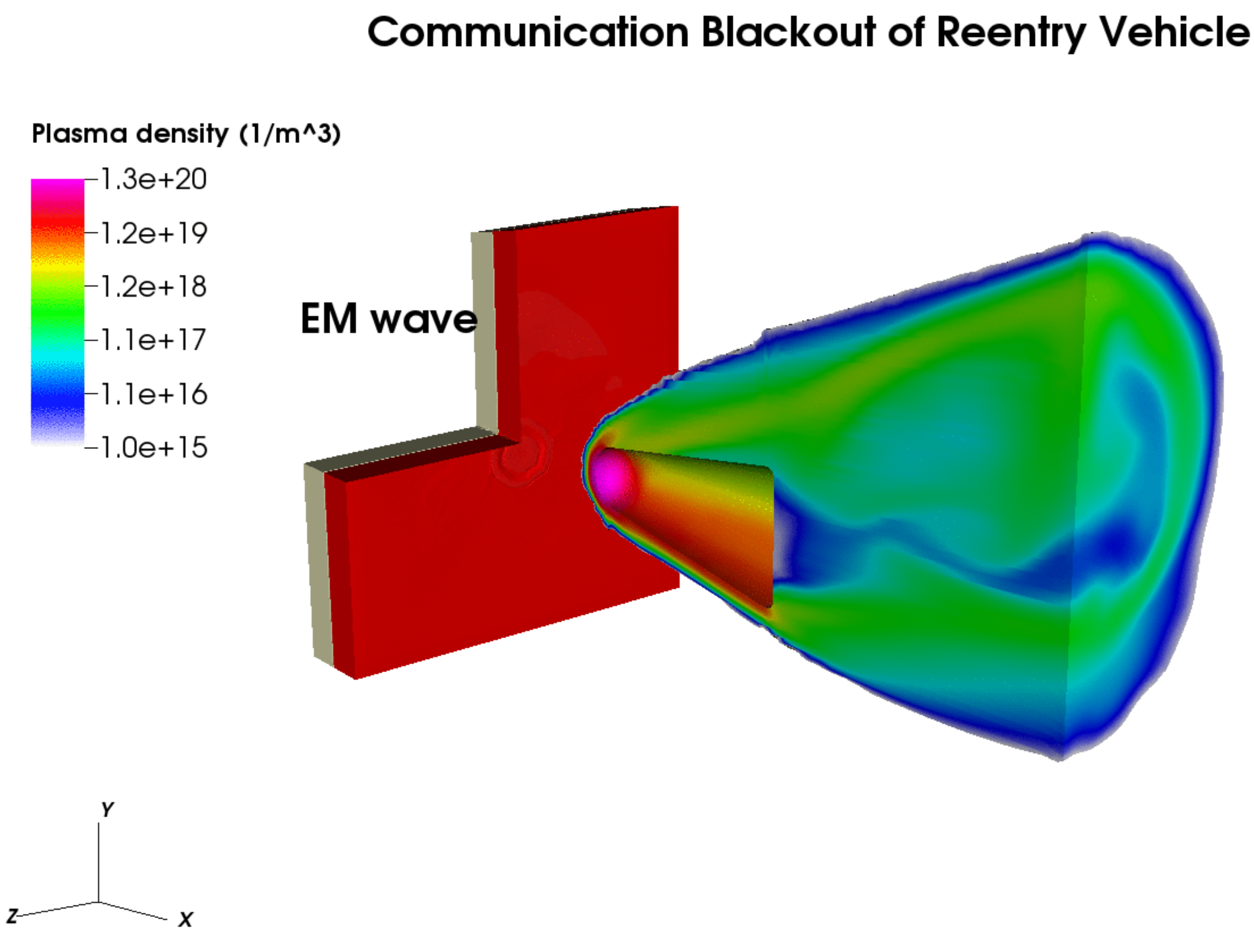


Ez (V/m)



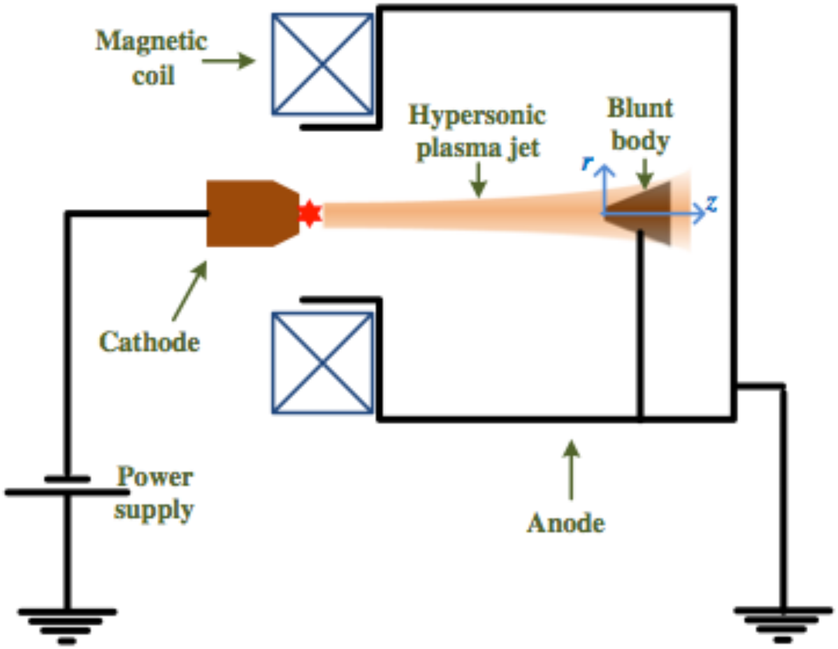
RAMC at an AOA

15° AOA, Mach 24, 7 species chemistry



Experiment to Simulate High Speed Plasma Flow Over a Blunt Cone: Multi-fluids

GWU MpNL
experiment setup



Arc current: 150 A

Arc Voltage: ~35 V

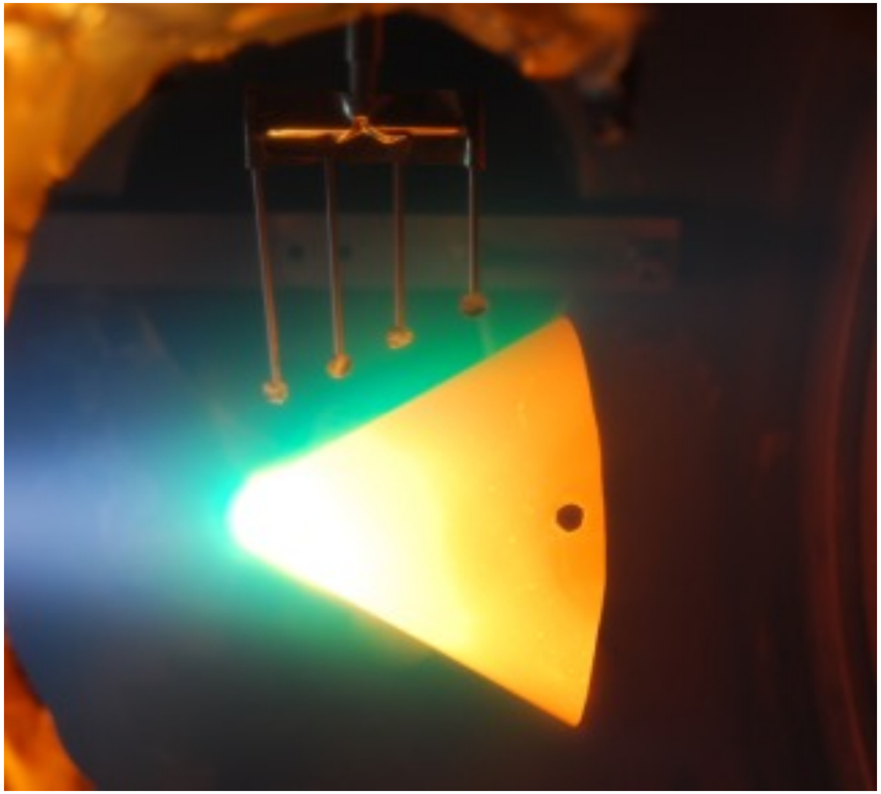
Cathode material: Cu

Cathode Diameter: 0.5 inch

Anode: Chamber

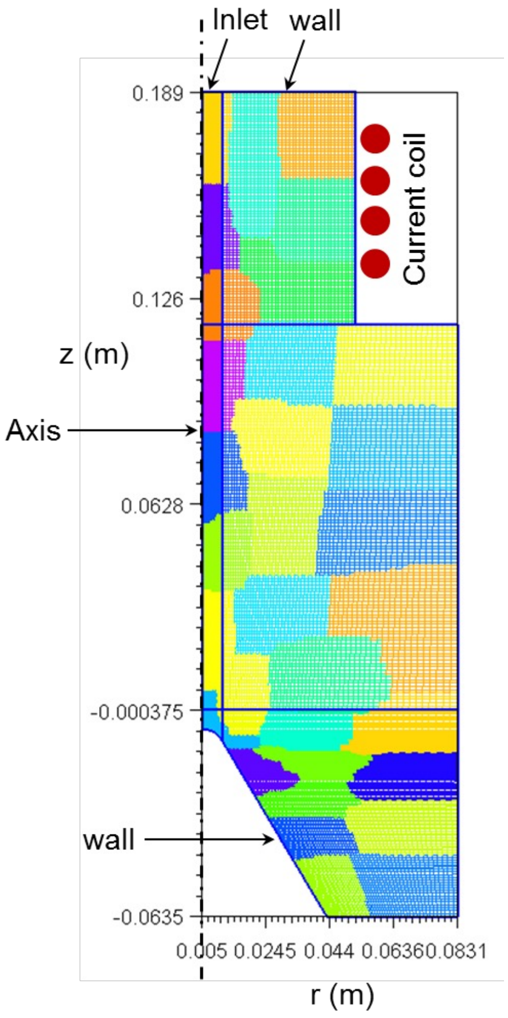
B at coil center: ~370 Gauss

Plasma flow over Mo cone



Experiments designed by Alexey Shashurin and Michael Keidar at GWU

Grid Partition

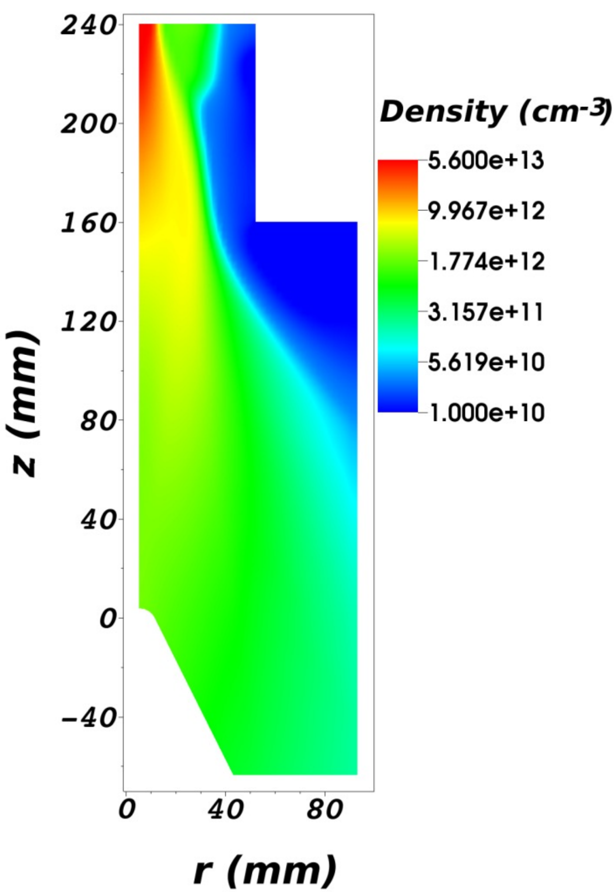


Gmsh or Cubit can be used for the unstructured mesh generation.

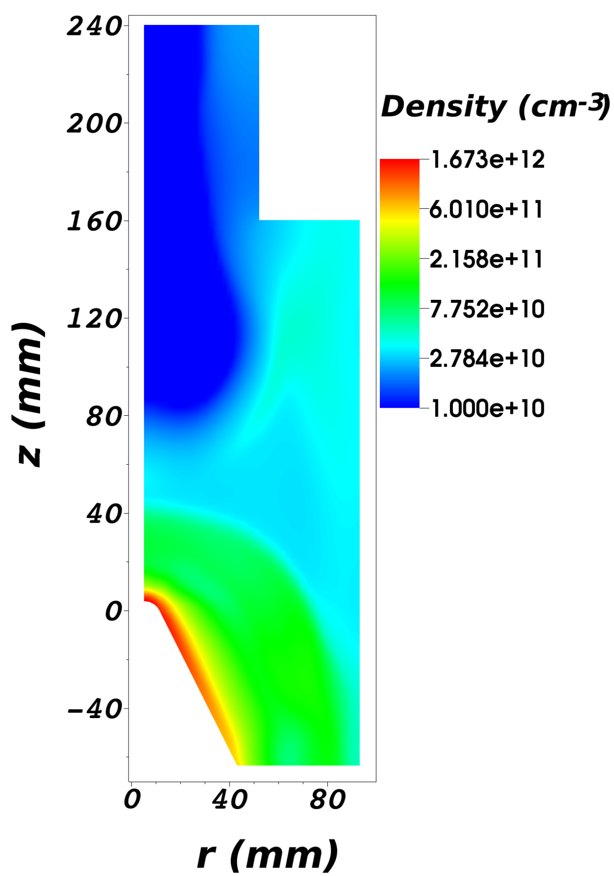
External magnetic field is imposed using current carrying coils.

Multi-Fluid Simulation

Plasma jet flow

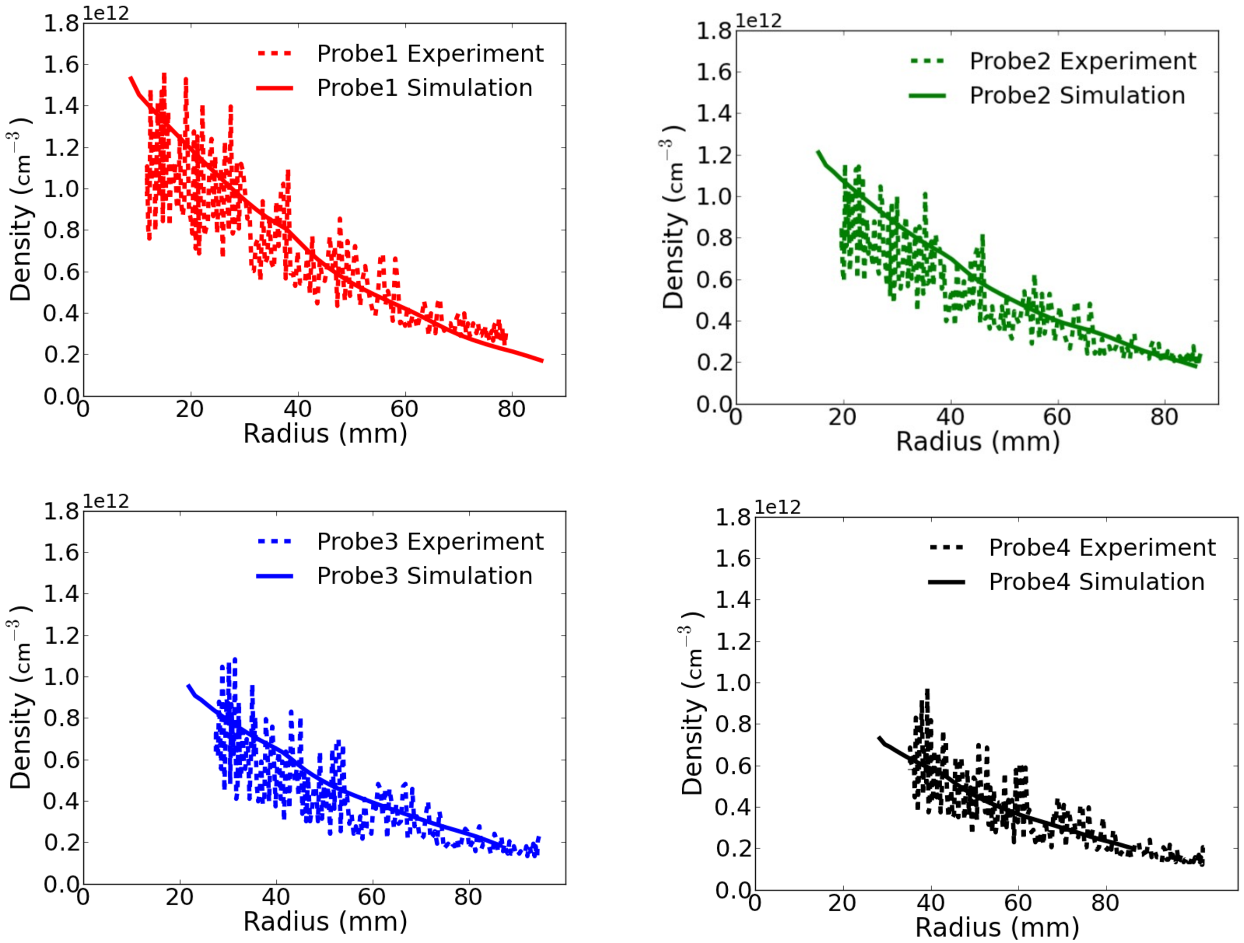


Re-evaporation



- Two fluids two equation simulation
- Gas dynamic MHD equations for jet plasma
- Euler equations for re-evaporated material
- Unstructured grid parallel simulation

Multi-Fluids Comparison



The radial distribution of electron density measured at 4 equally spaced locations on the lateral surface of cone is compared.

The simulation results match well with the measurements.

A. Shashurin,T. Zhuang,G. Teel,M. Keidar,M. Kundrapu,J. Loverich,I. I. Beilis, andY. Raitses. "Laboratory Modeling of the Plasma Layer at Hypersonic Flight". Journal of Spacecraft and Rockets, (2014)

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Summary

- USim is a good candidate for modeling hypersonic flows with multiphysics, particularly electromagnetics.
- Flexible equation system is a big advantage of USim.

USim Examples

Diffusion

Supersonic crossflow over a cylinder

Flow over a cylindrical rod

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Thank you

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Summary

- USim unstructured multi-species and multi-fluid models were presented
- Electron densities in the multi-species model showed similar trends as measured in the original experiment of RAM C.
- The higher densities predicted in the simulation may match experiments if radiation losses and diffusion are considered.
- Multi-fluid model was verified using analytical solutions from the dispersion relations and the collisional interaction was validated the experiments. Both comparisons showed an excellent agreement.
- Whistler mode wave propagation in the plasma was demonstrated on RAM C reentry vehicle.

Fluxes

General Equation

$$\frac{\partial q}{\partial t} + \nabla \cdot F\left(q, v_1, ...v_n\right) = \psi\left(q, v_1, .., v_n\right) + S$$

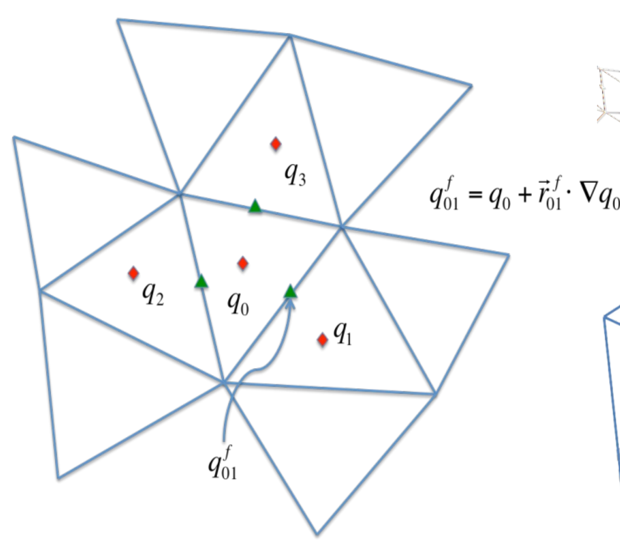
Least Squares Polynomial

$$q\left(x, y\right) = a_0 + a_1 x + a_2 y + a_3 x\, y + a_4 x^2 + a_5 y^2$$

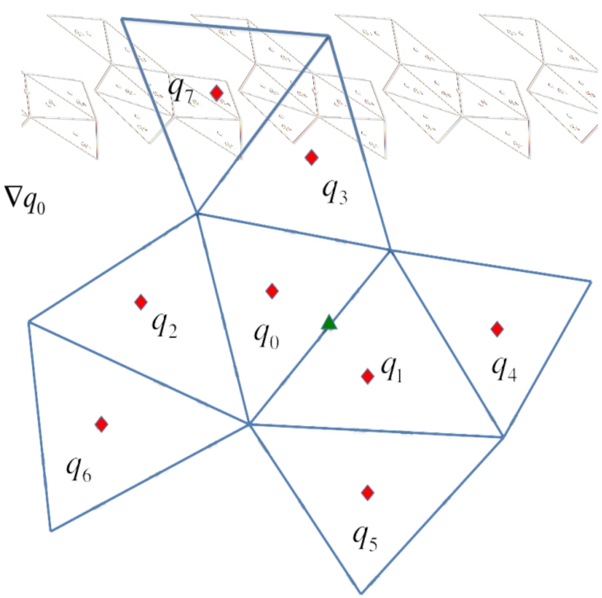
$$\nabla q\left(x, y\right) = \begin{bmatrix} a_1 + a_3 y + a_4 2x \\ a_2 + a_3 x + a_5 2y \end{bmatrix}$$

$$\sum_{k=1}^{N} \frac{A_k}{V} F_k \cdot n_k$$

Advection Terms



Diffusion Terms



HLLE, HLLC,Lax flux schemes
Positivity preserving sheme allows upto 10^9 times pressure and density jumps

USim: Solvers

Time integrators

1-4th order Runge-Kutta

- Used for integrating most of the equations in USim

Super Time Stepping

- Can be used up to 1000 times smaller time steps.
- Used for integrating the sources

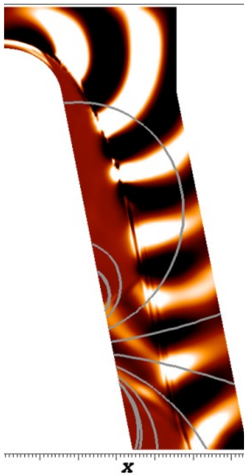
Local ODE Integrator from Boost library

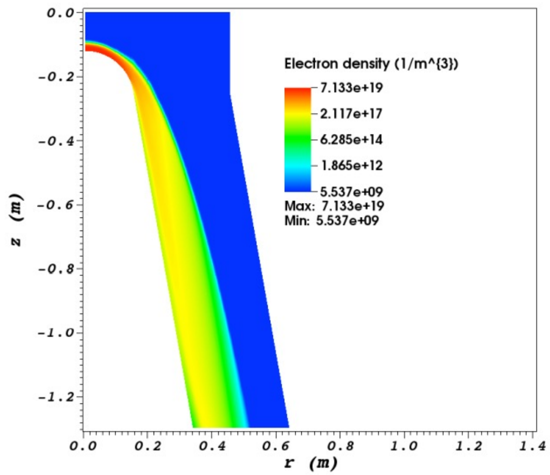
- Used for integrating the reaction rate equations

Iterative

JFNK methods from Trilinos

- Used for the Poisson equation
- Implicitly solving the hyperbolic equations.





USim solvers allow multi-scale multi-physics simulations with varying time scales:

10^{-7} Flow speeds

10^{-9} Reactions

10^{-11} EM wave frequency

10^{-14} Plasma frequency, cyclotron frequency

Computational Scheme

$$\frac{\partial}{\partial t}\begin{pmatrix} \rho \\ \rho\, u_x \\ \rho\, u_y \\ \rho\, u_z \\ e \end{pmatrix} + \nabla\cdot\begin{pmatrix} \rho\, u_x & \rho\, u_y & \rho\, u_z \\ \rho\, u_x^2 + P & \rho\, u_x\, u_y & \rho\, u_x\, u_z \\ \rho\, u_y\, u_x & \rho\, u_y\, u_y + P & \rho\, u_y\, u_z \\ \rho\, u_z\, u_x & \rho\, u_z\, u_y & \rho\, u_z\, u_z + P \\ u_x\,(e+P) & u_y\,(e+P) & u_z\,(e+P) \end{pmatrix} = \begin{pmatrix} S_i \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

- MUSCL reconstruction of variables: compute gradient at the cell center and extrapolate on to the faces
- Limiter to avoid spurious oscillations (min-mod, mc, superbee, Van-Leer)
- Flux schemes to get interface fluxes (HLLE, HLLC, Lax)
- Diffusion fluxes: least squares gradient on the faces and then integrate
- Runge-Kutta time integration
- Super time stepping
- Boost ode integrators

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Mach 24
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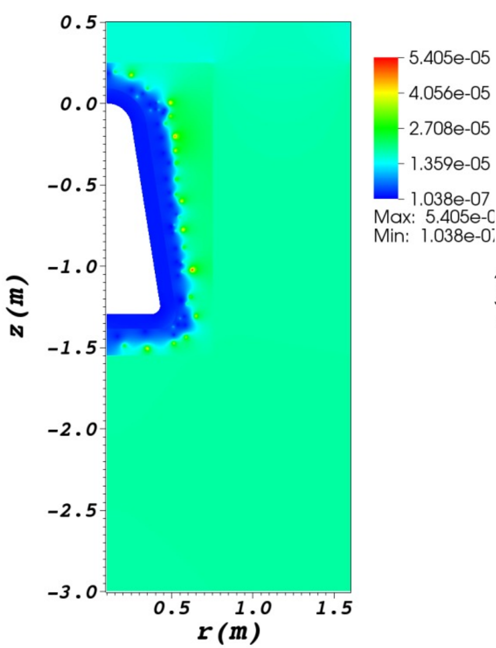
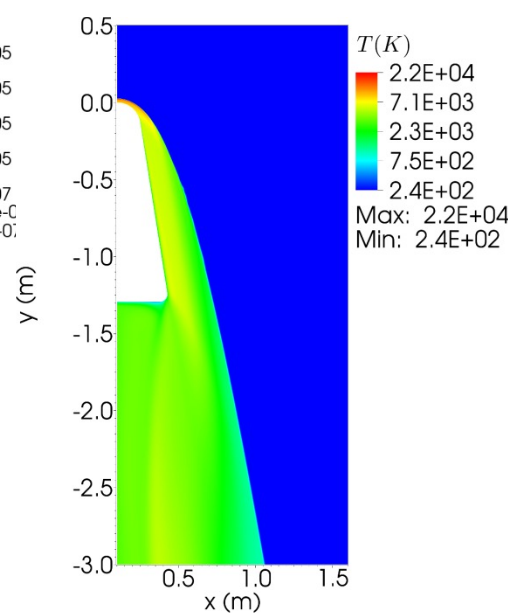
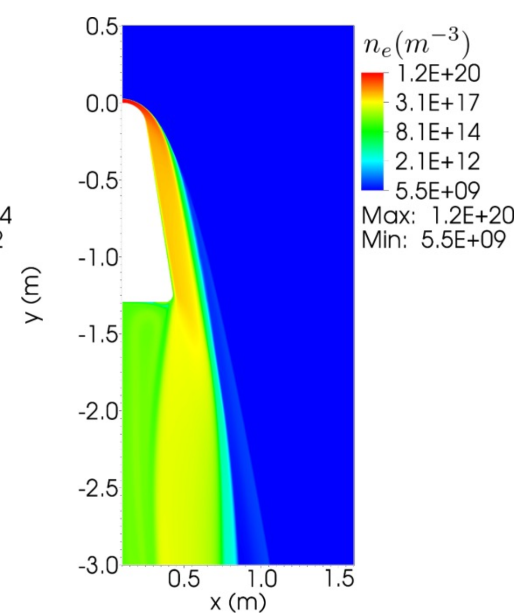
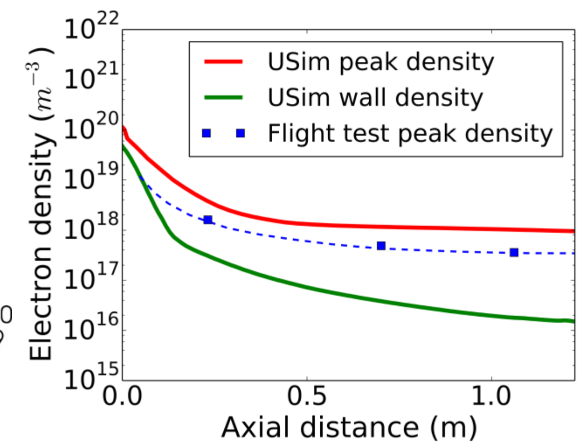


Temperature (K)



Electron density (m^{-3})





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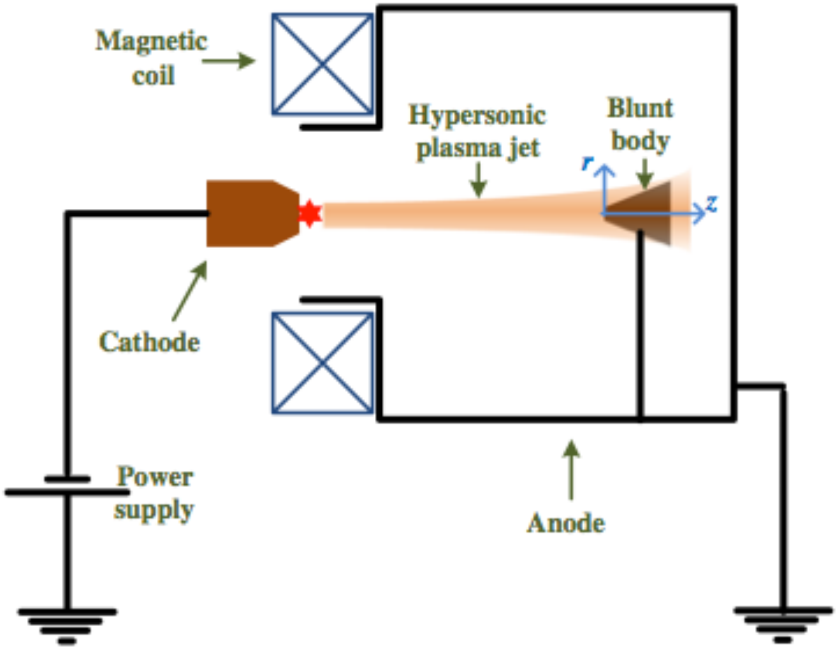

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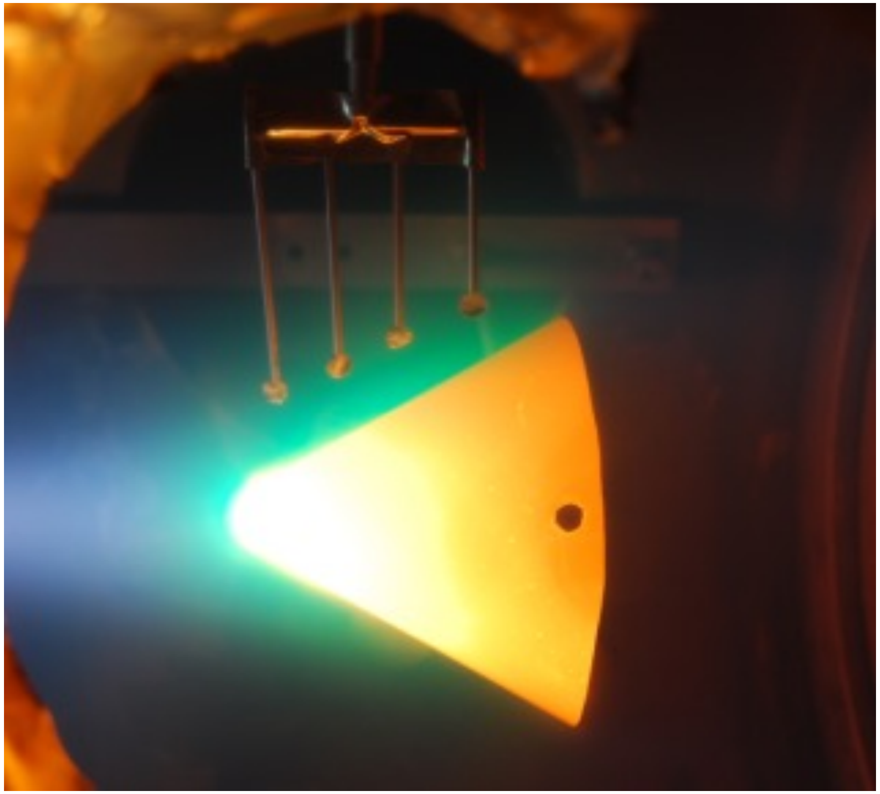
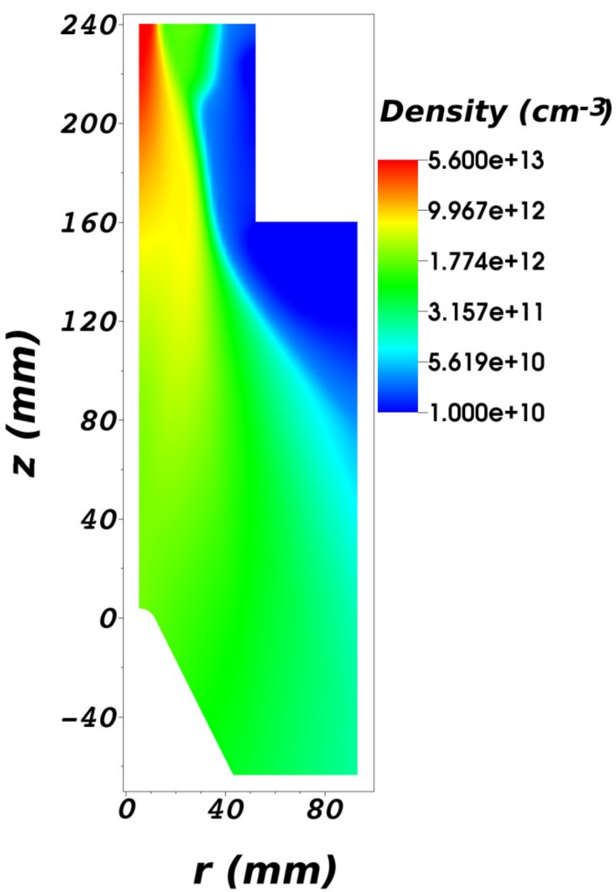
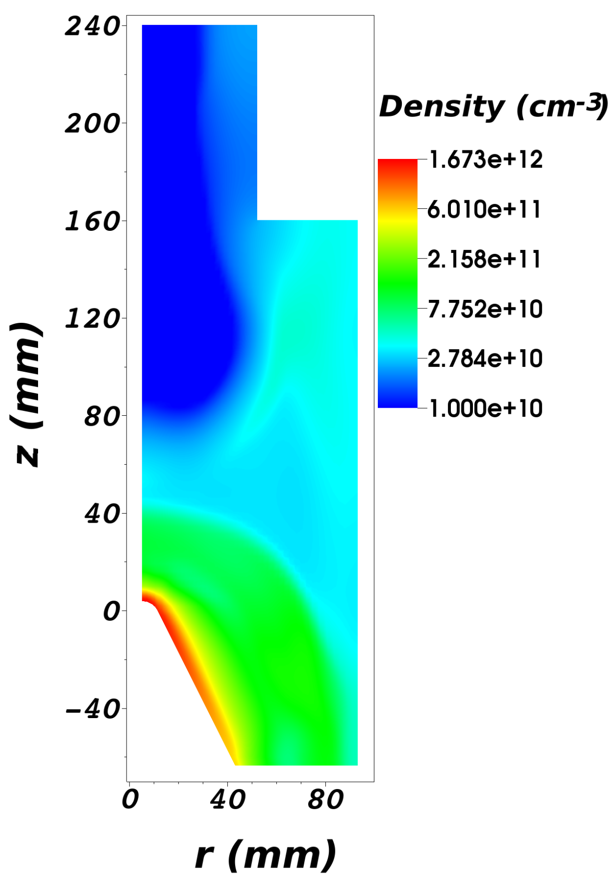


Multi-Fluid Simulation

Plasma jet flow



Re-evaporation



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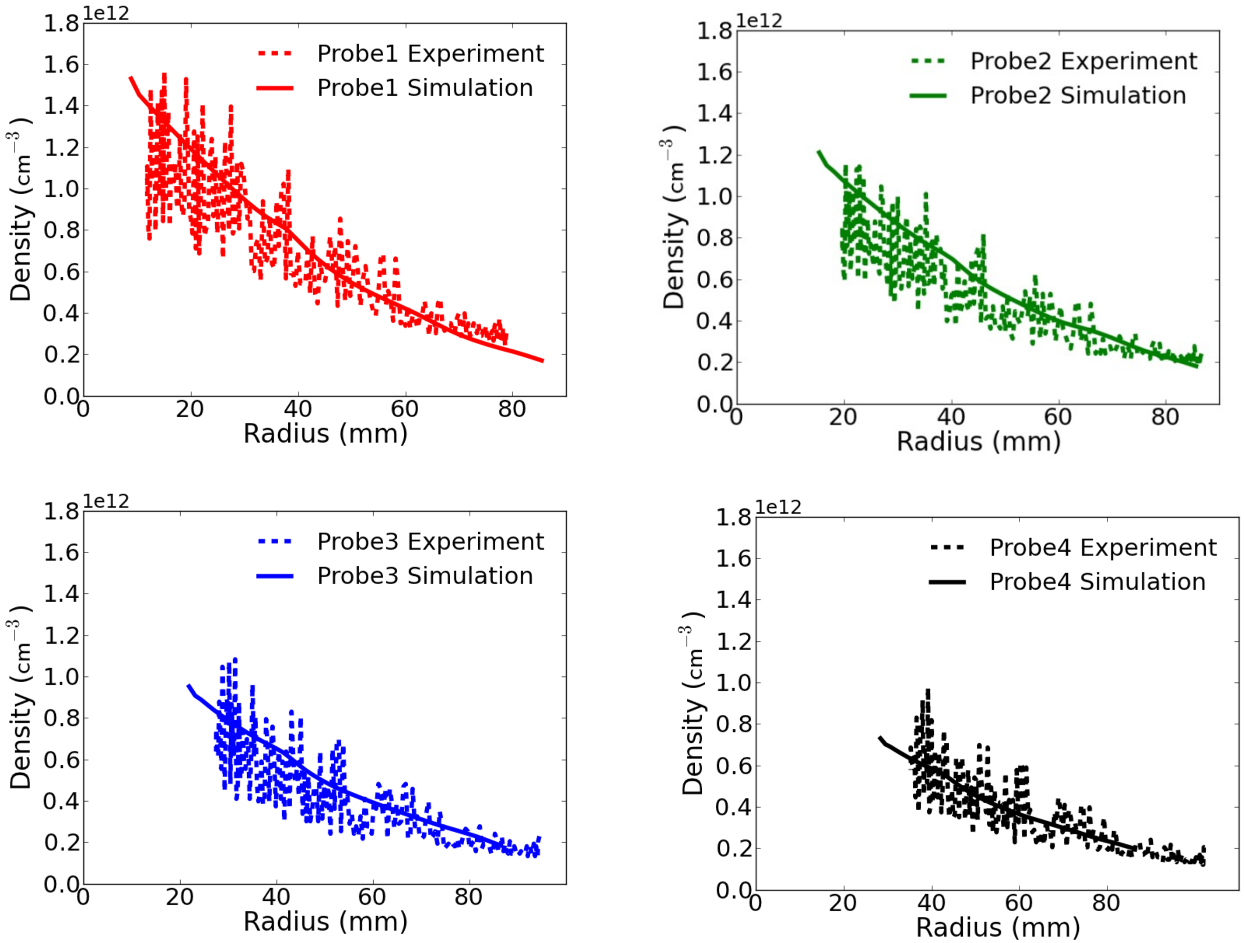

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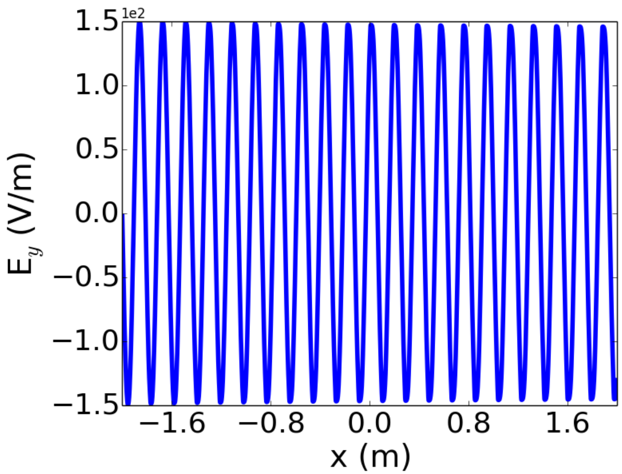
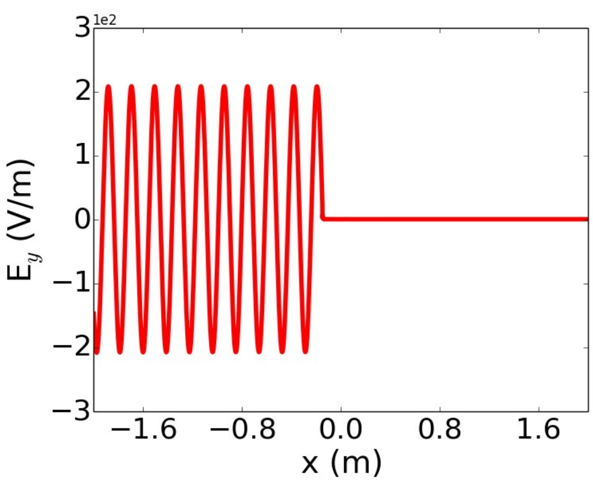
Whistler Wave Propagation

f = 1.6 GHz; $E_y = 150\sin(\omega t)$

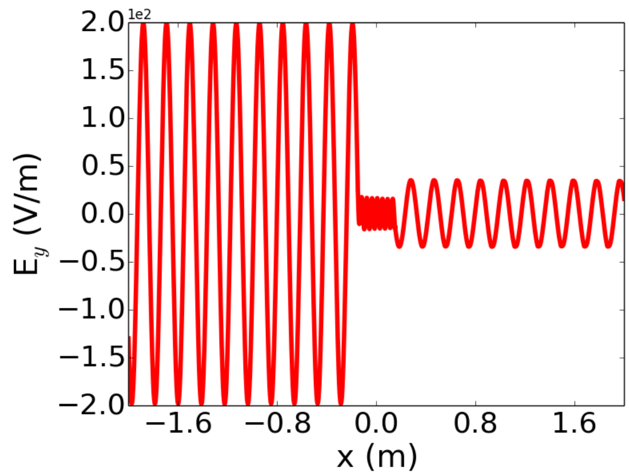


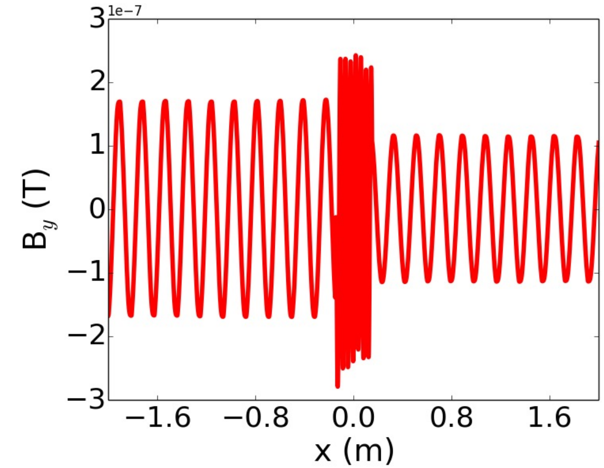
Plasma slab = 0.3 m

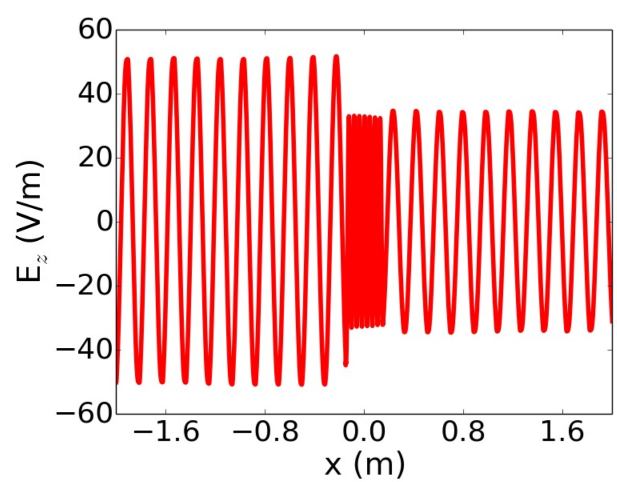
Density = 10^{19} m^{-3}

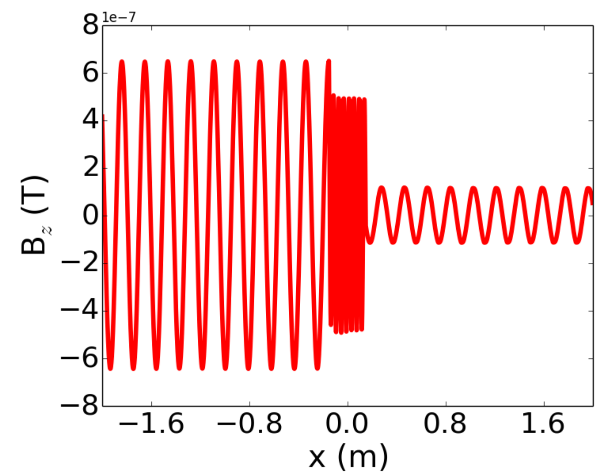


$B0_x = 1$ T



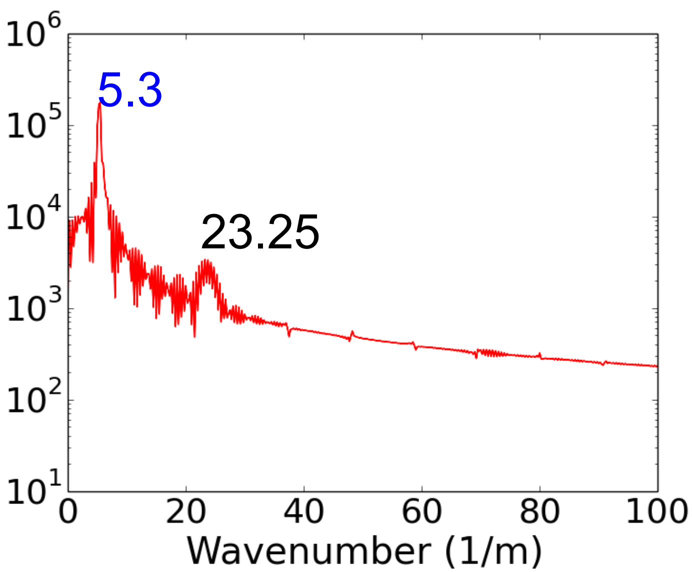






Dispersion Relation Comparison

Fourier transform of E_y in space



$n_e (m^{-3})$	B0 (T)	k (m^{-1}) analytical	k (m^{-1}) USim
1e18	0.1	34.97	No clear peak
1e18	0.2	19.66	19.38
1e18	0.4	13.32	13.01
1e19	0.5	34.40	34.01
1e19	1.0	23.89	23.25
1e19	2.0	17.07	16.50

- Dispersion matrix has to be sol… Whistler wave number. Mather…
- Two-fluid solver results match well with the analytical solution.

TECH-X
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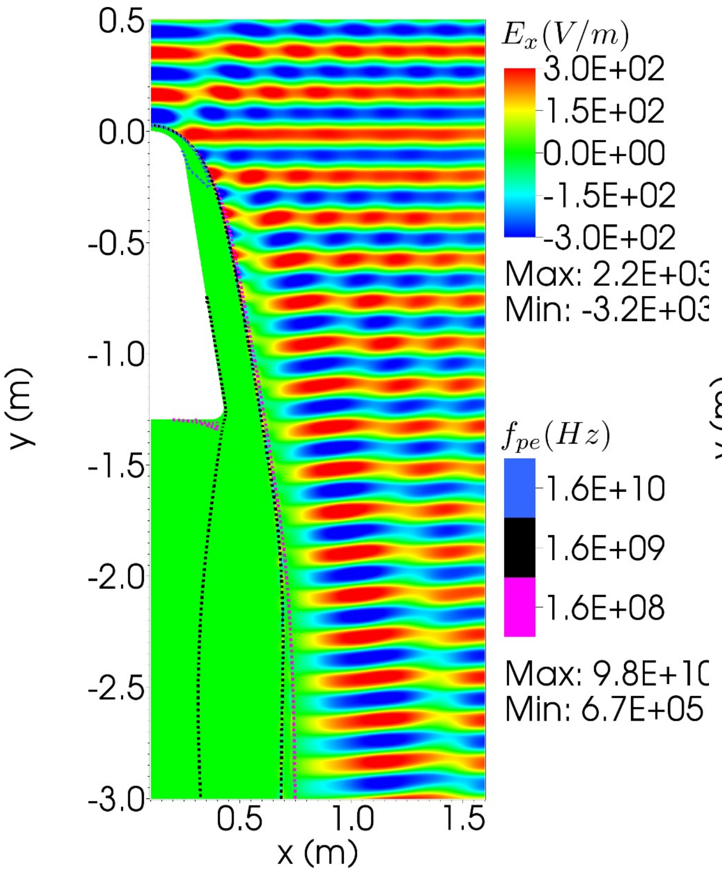
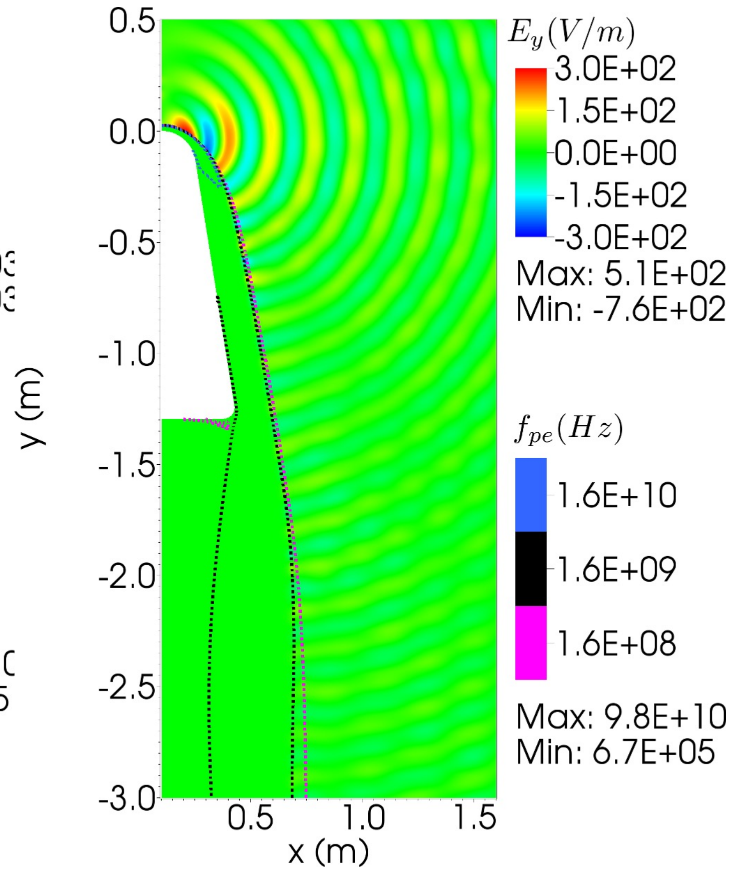
Wave Propagation on RAMC

f = 1.6 GHz; $E_x = 300\sin(\omega t)$; plasma density > 10^{19} m^{-3}





EM wave was fully reflected by the plasma layer.

Electrophilic fluid injection

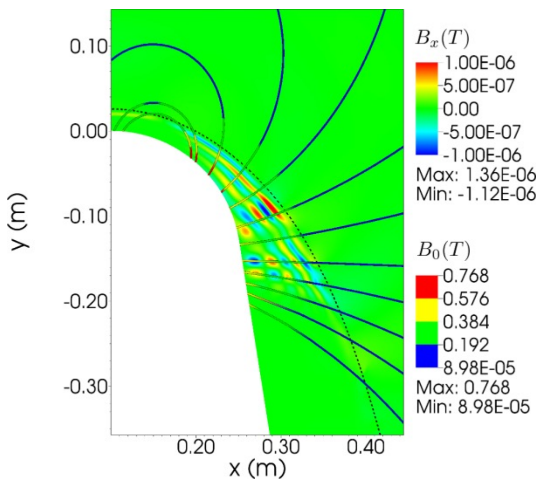
Reduced bit rate based on the reflected wave

Magnetic window

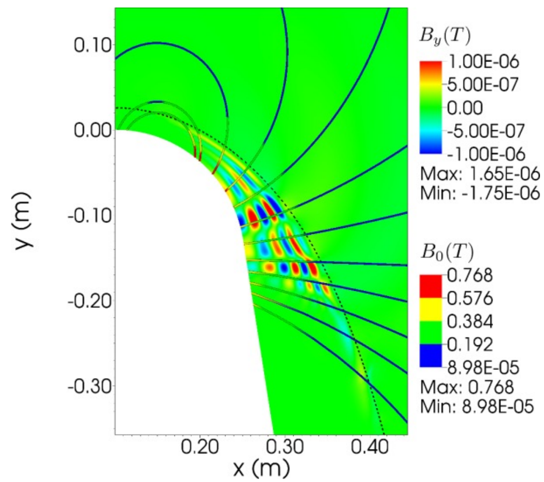
Magnetic Window on RAM C

A magnetic field of 0.7T was applied near the surface!
Wave propagates on to the surface

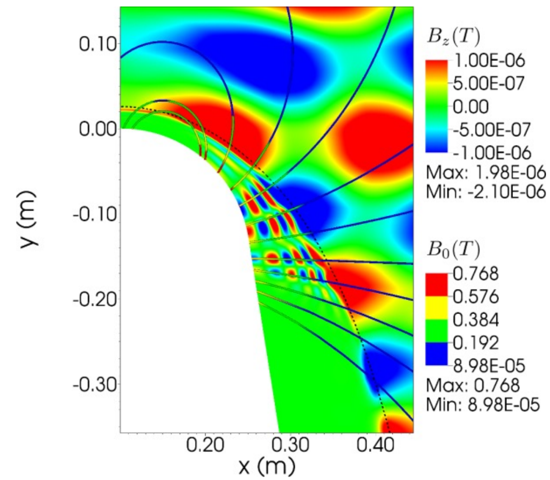
Bx (T)



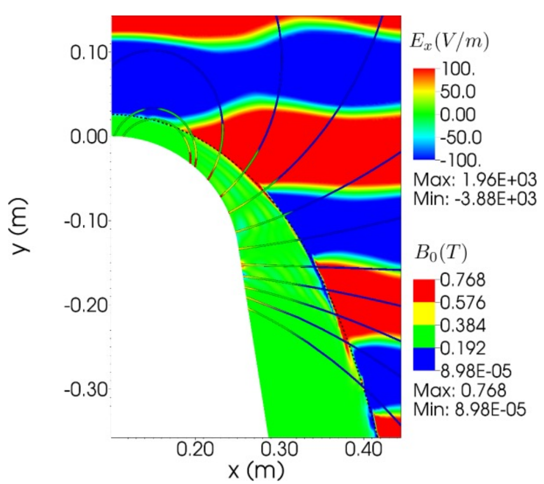
By (T)



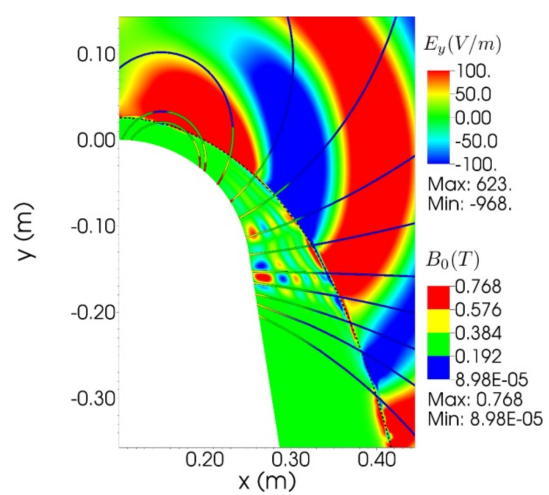
Bz (T)



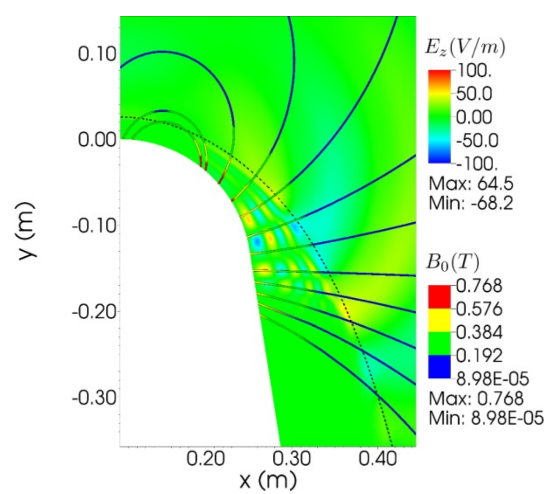
Ex (V/m)



Ey (V/m)

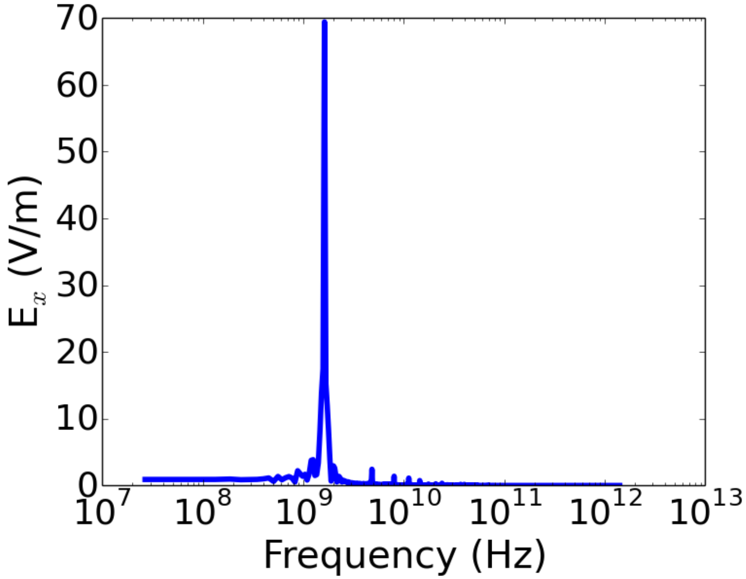


Ez (V/m)



Magnetic Window on RAM C: Wave Analysis

EM wave frequency on the surface

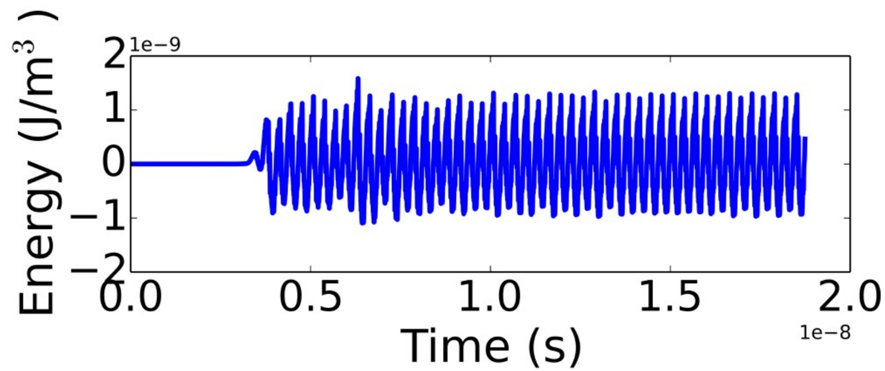


EM wave frequency is presreserved after propagating on to the surface in the whistler mode.

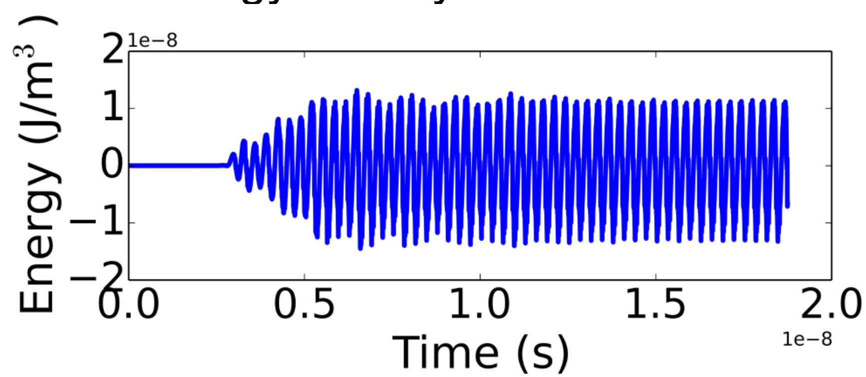
EM wave's energy density is about 25% of the actual wave!

$$Q_{EM} = \frac{1}{2}\left(\epsilon_0 \vec{E}\cdot\vec{E} + \frac{1}{\mu_0}\vec{B}\cdot\vec{B}\right)$$

Energy density outside the plasma layer

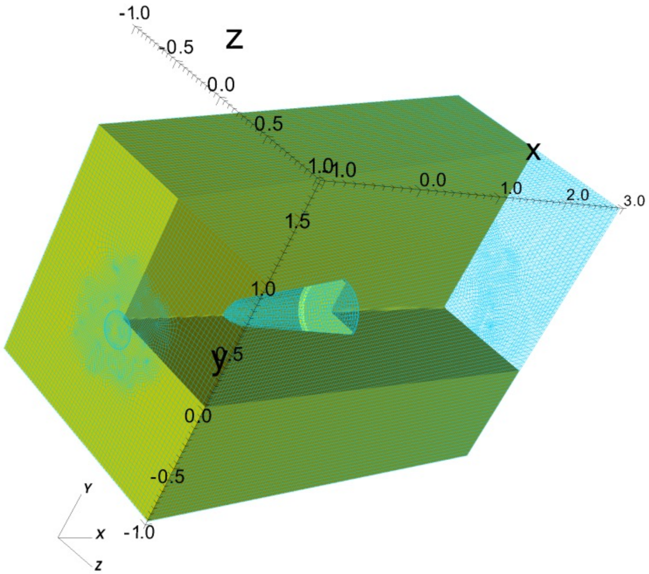


Energy density at the surface

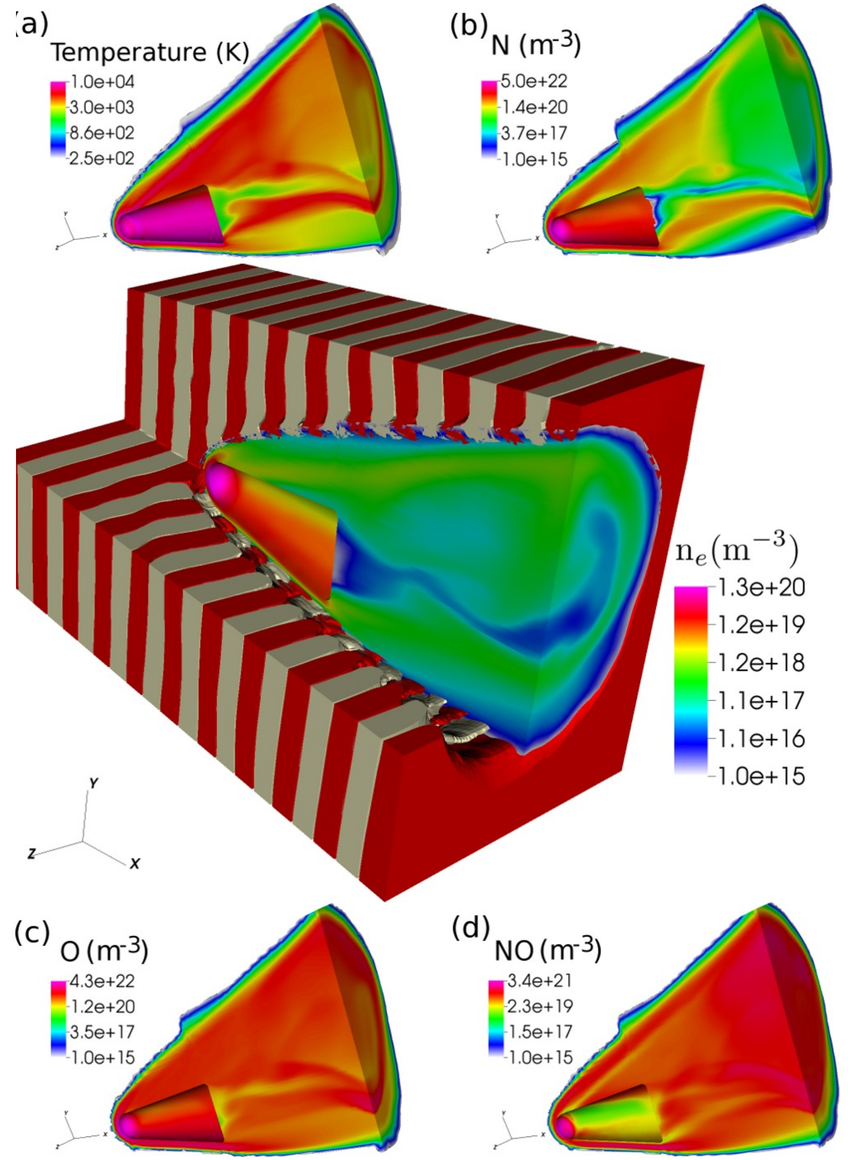


RAM C at an AOA

853,000 hexahedral cells minimum and maximum edges of 7 mm and 4 cm. 2x2 m^2 front and 2x2.75 m^2 rear. Length 4 m.



AOA = 15◦
Altitude = 61 km
Mach number = 23
EM wave at 0.8 GHz

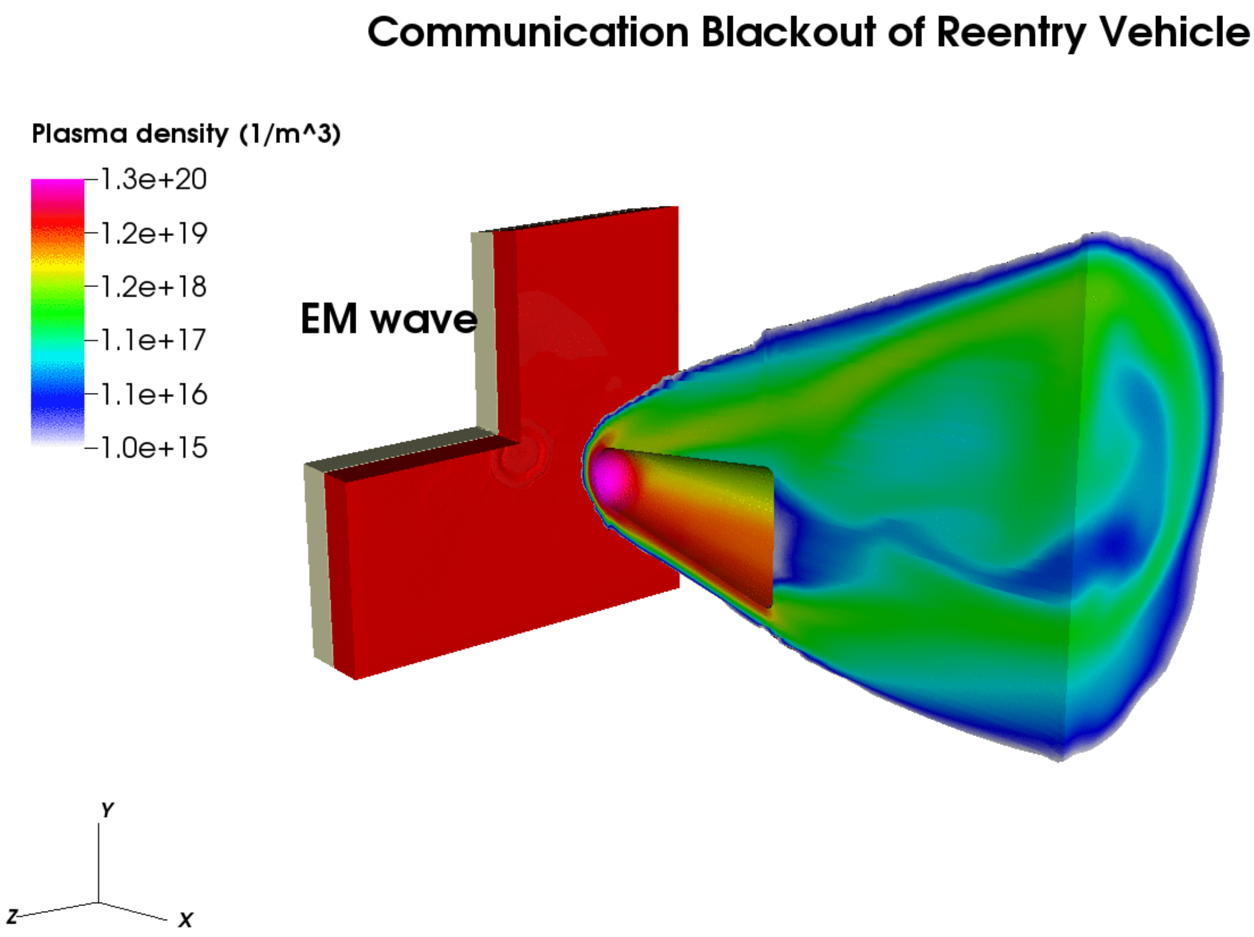


The peak density of the plasma is 1.3×10^20 m−3 .
The peak temperature of the gas is 10470 K.
N and O are 5×10^22 and 4.3×10^22 m−3.

EM wave is fully reflected at n_e = 8x10^15 m^-3

RAMC at an AOA

15° AOA, Mach 24, 7 species chemistry



SIMULATIONS EMPOWERING YOUR INNOVATIONS

Euler Three Temperature Model

$$\frac{\partial}{\partial t}\begin{pmatrix} \rho_l \\ \rho\, u_x \\ \rho\, u_y \\ \rho\, u_z \\ e_t \\ e_e \\ e_v \end{pmatrix} + \nabla\cdot\begin{pmatrix} \rho_l\, u_x & \rho_l\, u_y & \rho_l\, u_z \\ \rho\, u_x^2 + P & \rho\, u_x\, u_y & \rho\, u_x\, u_z \\ \rho\, u_y\, u_x & \rho\, u_y\, u_y + P & \rho\, u_y\, u_z \\ \rho\, u_z\, u_x & \rho\, u_z\, u_y & \rho\, u_z\, u_z + P \\ u_x\,(e_t + P) & u_y\,(e_t + P) & u_z\,(e_t + P) \\ u_x\,(e_e + P_e) & u_y\,(e_e + P_e) & u_z\,(e_e + P_e) \\ u_x\,(e_v) & u_y\,(e_v) & u_z\,(e_v) \end{pmatrix} = \begin{pmatrix} s_l \\ 0 \\ 0 \\ 0 \\ \mathbf{E}\cdot\mathbf{J} + Q_{v-t} \\ \mathbf{E}\cdot\mathbf{J_e} \\ -Q_{v-t} \end{pmatrix}$$

$$P = P_l + P_e$$
$$Q_{v-t} = \sum_l \rho_l \left(\frac{e^*_{v,l} - e_{v,l}}{\tau_l}\right)$$
$$e_{v,l} = \frac{R}{M_l}\left[\frac{\theta_{v,l}}{exp(\theta/T_v) - 1}\right]$$
$$e^*_{v,l} = \frac{R}{M_l}\left[\frac{\theta_{v,l}}{exp(\theta/T) - 1}\right]$$
$$J_{e,x} = n_e e^- u_x$$
$$J_x = (n_i - n_e)\, e^- u_x$$
$$E_x = -\frac{\partial P_e}{\partial x}\frac{1}{n_e e^-}$$

Θ is the characteristic vibrational temperature.

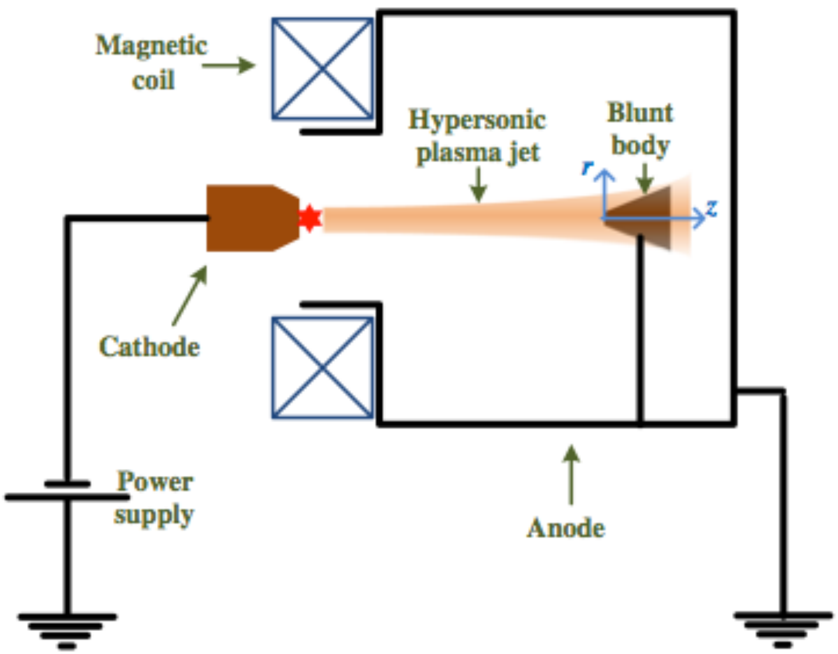
τ is vibrational-translational relaxation time

USim has a verified model of vibrational-translational relaxation. Verification ensures the correct implementation.

The electron energy evolution can be segregated to make the model strictly two-temperature.

Experiment to Simulate High Speed Plasma Flow Over a Blunt Cone: Multi-fluids

GWU MpNL experiment setup



Arc current: 150 A

Arc Voltage: ~35 V

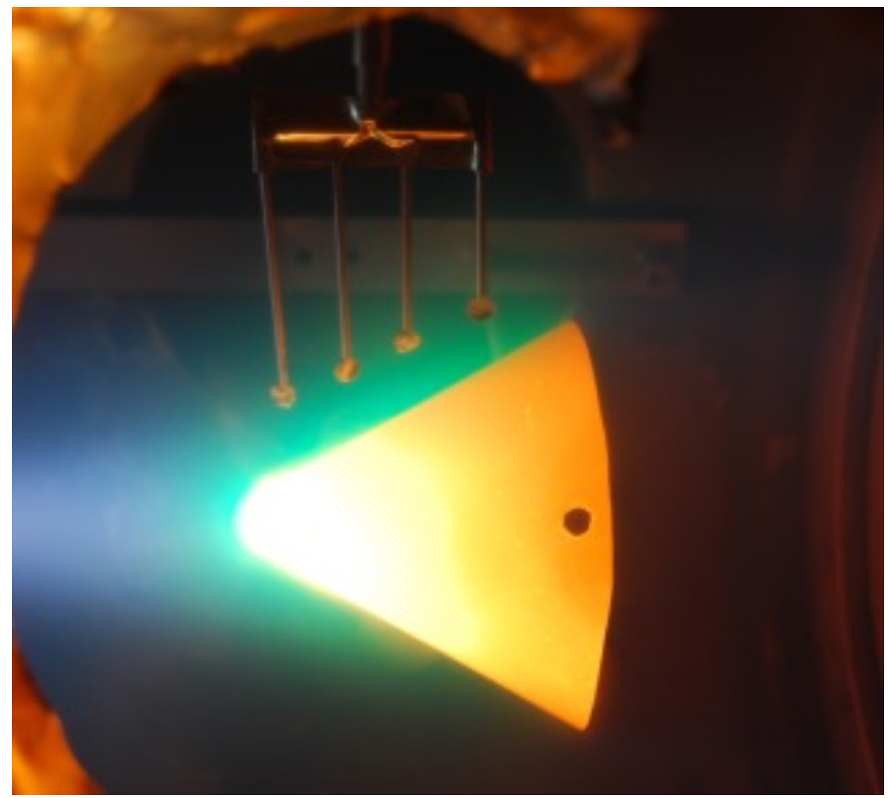
Cathode material: Cu

Cathode Diameter: 0.5 inch

Anode: Chamber

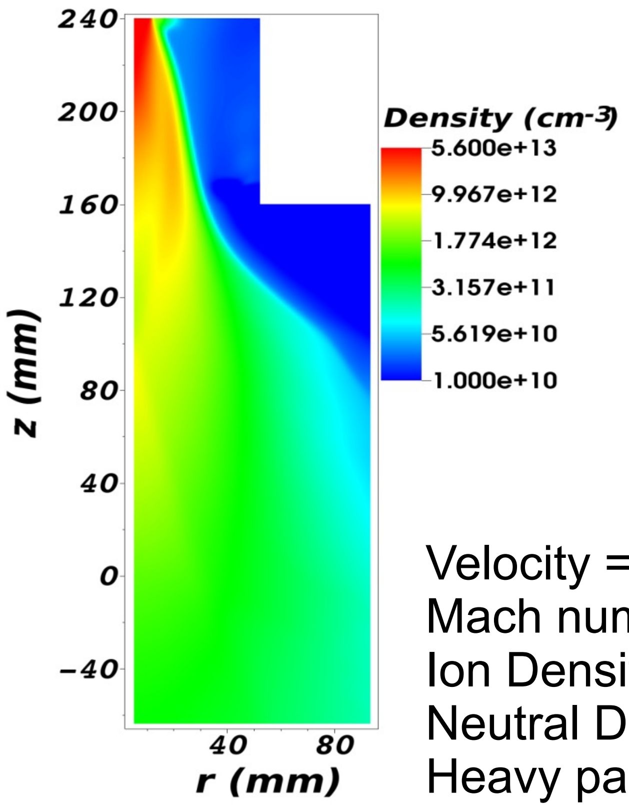
B at coil center: ~370 Gauss

Plasma flow over Mo cone

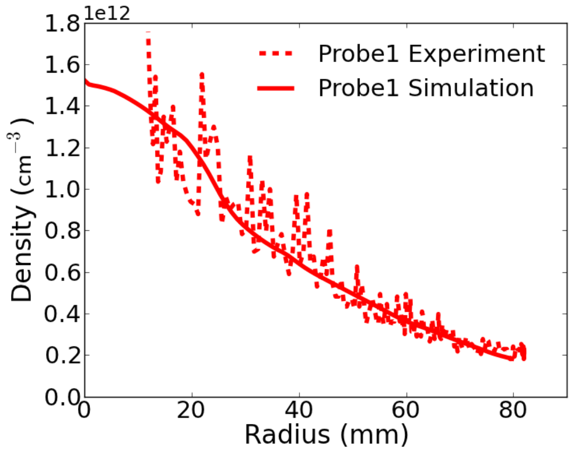


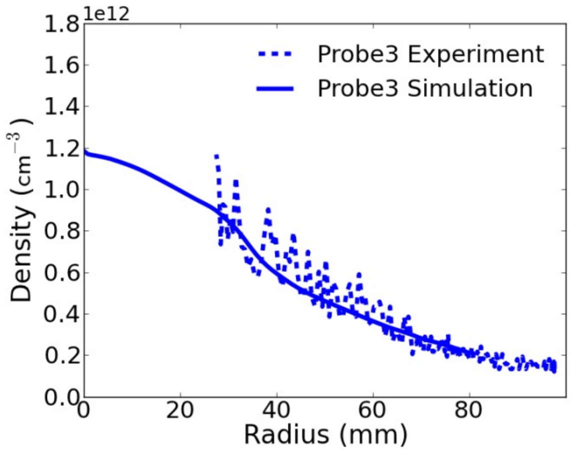
USim Validation -backup

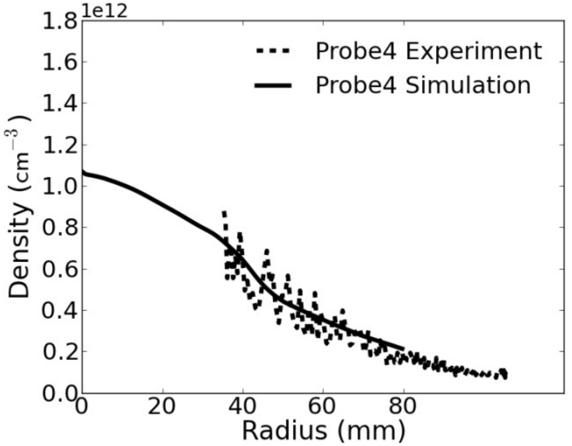
Plasma density of free jet



Comparison



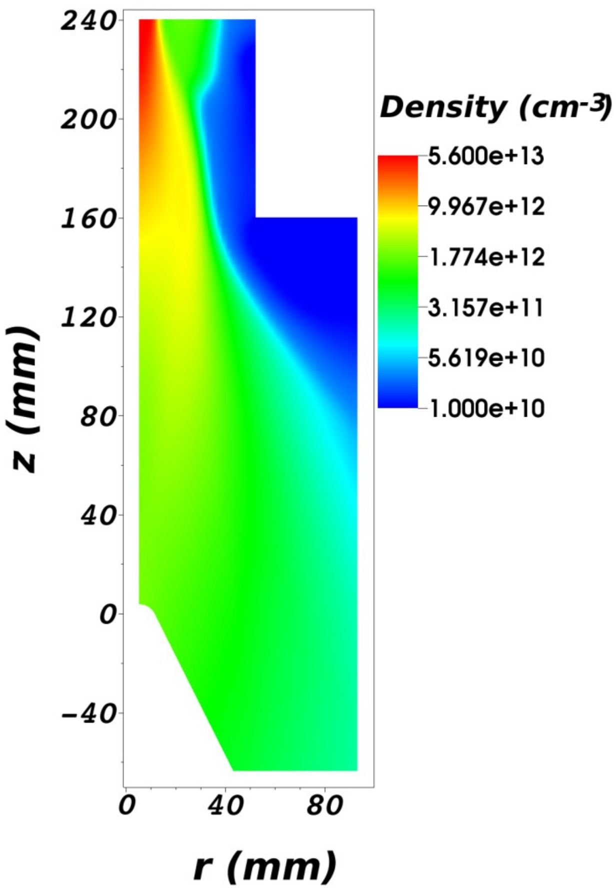




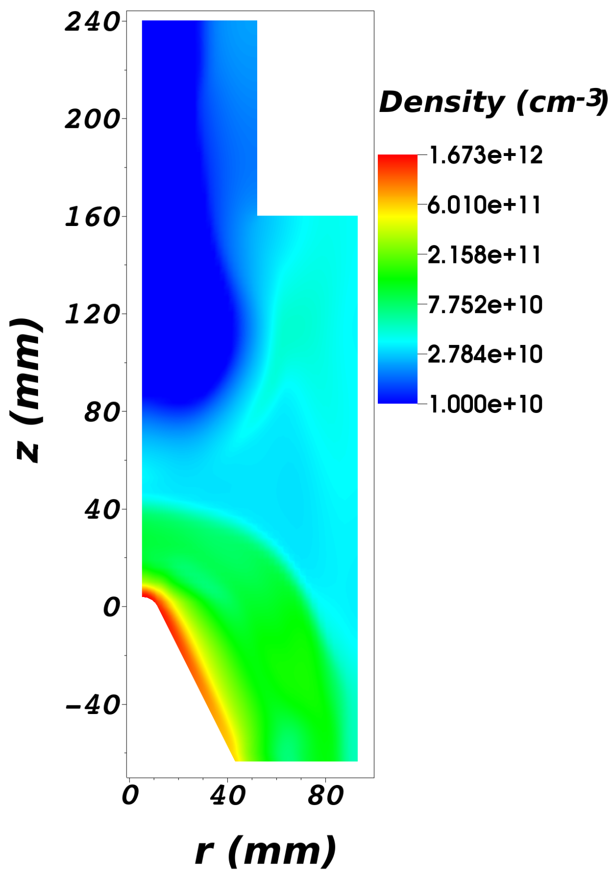
Velocity = 12800 m/s
Mach number = 4
Ion Density = 3.5e19 1/m^3
Neutral Density = 5.25e19 1/m^3
Heavy particleTemperature = 1.3 ev
Electron temperature = 2.5 ev
Charge number = 1.8

Multi-Fluid Simulation

Plasma jet flow



Re-evaporation



- Two fluids two equation simulation
- Gas dynamic MHD equations for jet plasma
- Euler equations for re-evaporated material
- Unstructured grid parallel simulation

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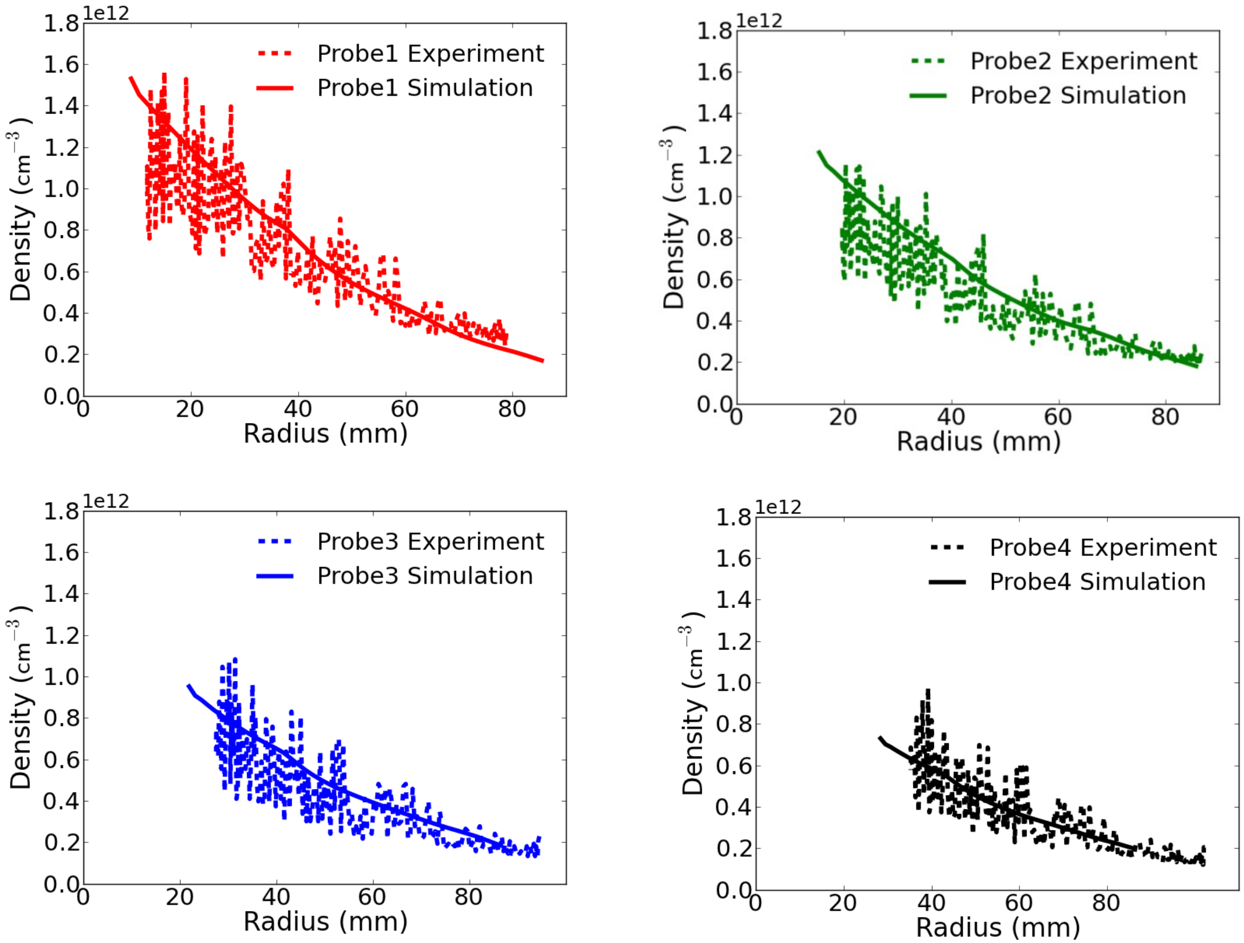
Multi-Fluids Comparison



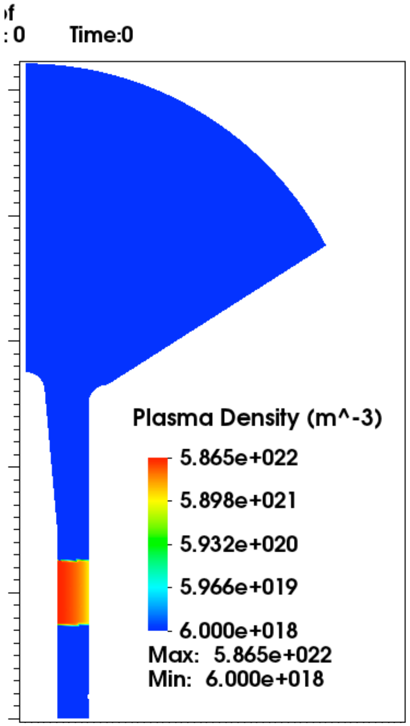
The radial distribution of electron density measured at 4 equally spaced locations on the lateral surface of cone is compared.

The simulation results match well with the measurements.

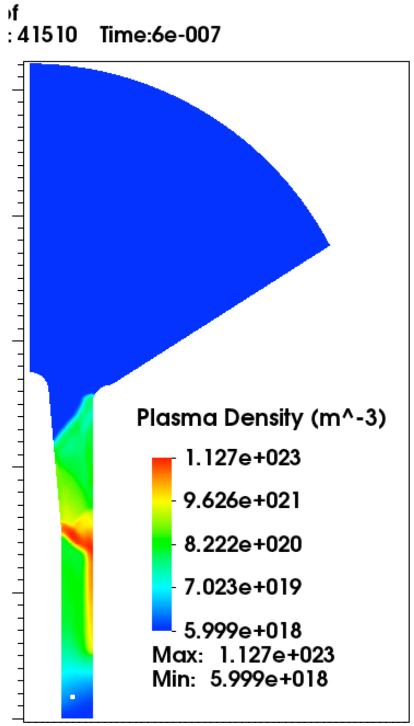
A. Shashurin,T. Zhuang,G. Teel,M. Keidar,M. Kundrapu,J. Loverich,I. I. Beilis, andY. Raitses. "Laboratory Modeling of the Plasma Layer at Hypersonic Flight". Journal of Spacecraft and Rockets, (2014)

USim modeling helped predict the density on axis for a given applied voltage

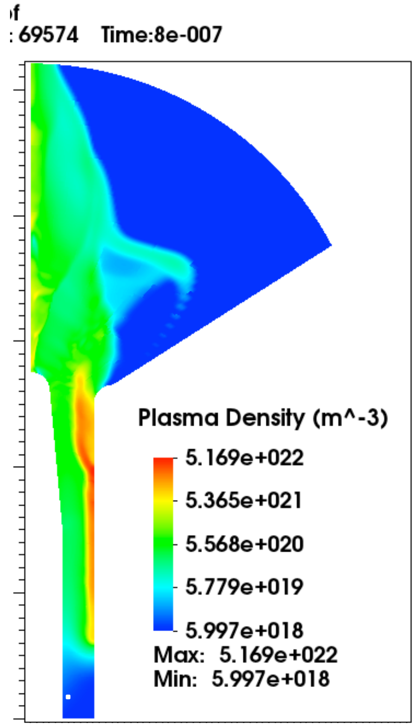
Initial distribution



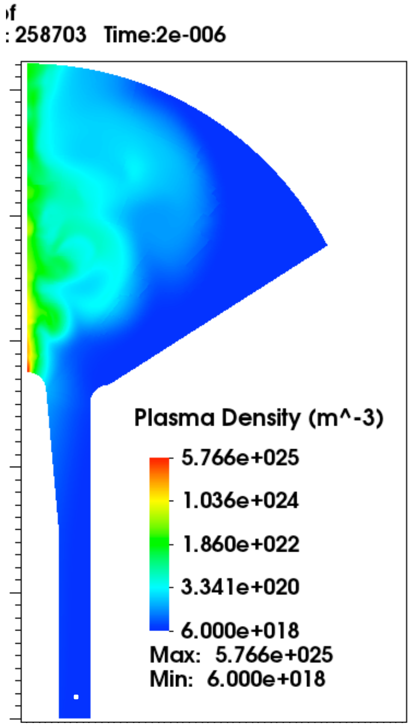
Plasma moves forward in the clearance



Plasma moves out of the clearance



Distribution at the end of the pulse



RAMC at an AOA

15° AOA, Mach 24, 7 species chemistry