An Improved Space Charge Limited Emission Algorithm in VSim

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Available in VSim 11!
For more details on this work, see the recent publication in Phys Plasmas

Available to everyone (no journal subscription required)!

A new simple algorithm for space charge limited emission

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Many devices in the directed energy community seek to operate at the highest possible power.

MILO - Magnetically Insulated Line Oscillator

A MILO converts DC input to rf output using a high current electron beam.

Researchers often want microwaves devices (like the MILO shown here) to operate at the highest possible power...and current!

But simulating ultra-high current can be challenging!
But let's start simple...just electron flow between two parallel plates

Potential between two plates in VSim

The 1D case is described by a theoretical expression, the Child-Langmuir equation

\[ J_{SCL} = \frac{4}{9} \epsilon_0 \sqrt{\frac{2eV^{3/2}}{m d^2}} \]

SCL = Space Charge Limit
CL = Child-Langmuir
(in 1D, steady state, they are the same)
Short interlude: the fascinating career of Katherine Blodgett

● First woman to get a PhD in physics from Cambridge
● First woman physicist hired by GE Research
● Generalized the 1D planar Child-Langmuir equation to cylindrical and spherical geometries (along with Langmuir)
● You will hear often of the Langmuir-Blodgett equation instead of the Child-Langmuir equation in complex geometries!
My speciality: what’s the wrong way to solve this problem?

- **First time step**: This problem is so easy! You know V and d. Just plug them in and emit that much current!

- **Second time step**: V and d didn’t change… Oh...shoot...wait...there’s already some current in my system!

- So OK...now calculate the current you already have and just emit however much current you need to get to $J_{SCL}$?

- **Third time step**: Oh boy! Now some current is actually going the wrong way. Do I include that? No? Yes?

- V and d still haven’t changed! I keep having to emit the same current even though there’s already current present.

\[ J_{SCL} = \frac{4}{9} \varepsilon_0 \sqrt{\frac{2eV}{m}} \frac{V^{3/2}}{d^2} \]
Better way: take advantage of the FDTD grid to make a natural SCL emission scheme!

Blow-up image of the first zone

\[
J_{\text{emit}} = \frac{4}{9} \varepsilon_0 \sqrt{\frac{2e}{m} \frac{E_0^3}{\sqrt{\Delta x}}}
\]
But is it strange for the current to depend on the cell size?

Doesn’t that mean better resolution will lead to diverging current?

No! Because E is going to zero at the surface, and the two effects cancel out.

The current is constant with changing resolution!
The 1D diode test shows the new algorithm works well!

The 1D diode reaches a steady state in just a few transit times.
As we improve the resolution, we get closer to the analytic answer.

The error converges like the number of cells to the $-2/3$ power, consistent with the fact the theoretical electron density scales as $2/3$ power.
The algorithm is robust enough to handle even more complicated cases, like secondary electron emission.

The electron phase space for the case where the anode is a perfect elastic electron reflector.

The current density at the anode and re-absorbed at the cathode. The simple estimate of steady-state current at the anode of $0.5 \, J_{CL}$ is a slight under-estimate due to virtual cathode formation.
The algorithm also captures accurately time-dependent voltage effects. A fast voltage rise can cause inductive spikes, and our algorithm captures that. A slower voltage rise can moderate those inductive spikes.
What about 2d? For instance, if the emission region has finite size

No analytic answer, but there are well-known numerical answers to which we can compare.

The current right in the center of the anode shows the same behavior as 1D.
The VSim results show a factor of three enhancement that matches previous well-known numerical results*.

The current density as a function of space after several transit times.

With benchmarking done, we are ready to apply this new algorithm to a MILO.

We model the MILO in full 3D with the geometry handled through the VSim embedded boundary capabilities.

The VSim SCL emission algorithm leverages this capability to work even in complex geometries.
A MILO converts DC to rf using the interaction of the electron beam with a rippled structure on the anode.

We will drive $V_{in}$ with a 500kV DC voltage, let the cathode emit electrons at the SCL value, and observe the output voltage.
We apply the new SCL emitter to the MILO

Applied Voltage

Resulting SCL particles (colored by velocity)

t=1 ns

t=3 ns

t=6 ns

Spoke formation is the sign of rf power!
The emitted and absorbed current of 50kA, in line with a 500kV device of this size agrees with Langmuir-Blodgett (when adjusted for relativistic effects).
The frequency of the rf is 13 GHz, matching previous results. The main frequency is 13 GHz, matching previous results. The presence of harmonics is another indication the SCL algorithm is working well.

The input DC voltage of 500kV is converted to rf voltage in the anode cavities

The main frequency is 13 GHz, matching previous results. The presence of harmonics is another indication the SCL algorithm is working well.

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