Modeling Phased Array Antennas

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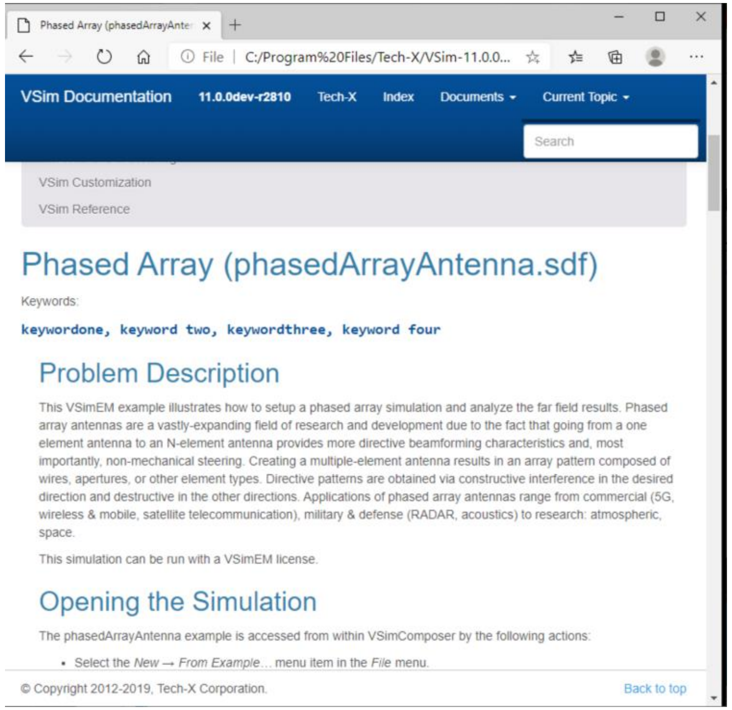
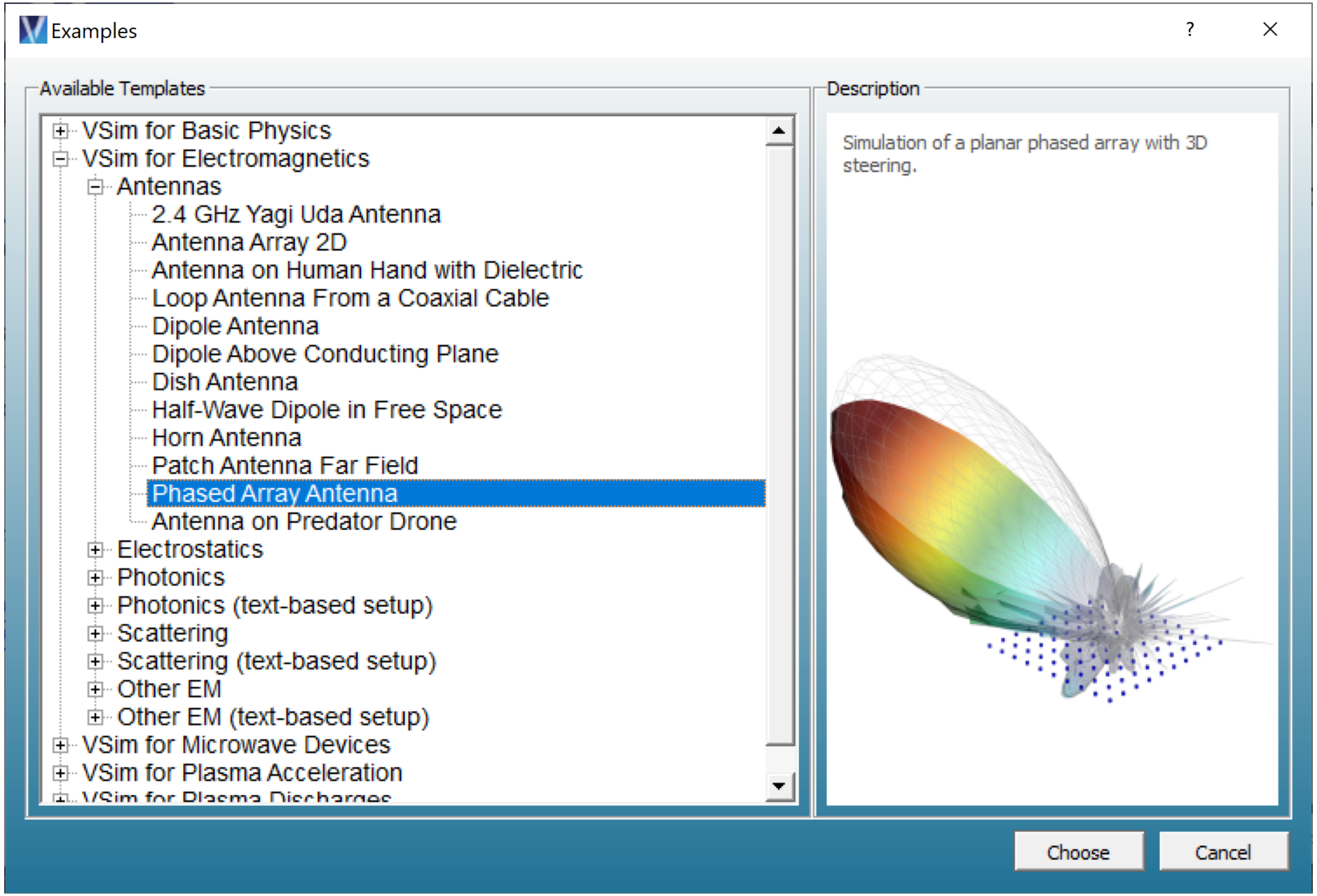
Tech-X SIMULATIONS EMPOWERING INNOVATION

TWSS 2020

We will go over the Phased Array Antenna example

- Available in VSim10
- In Documentation



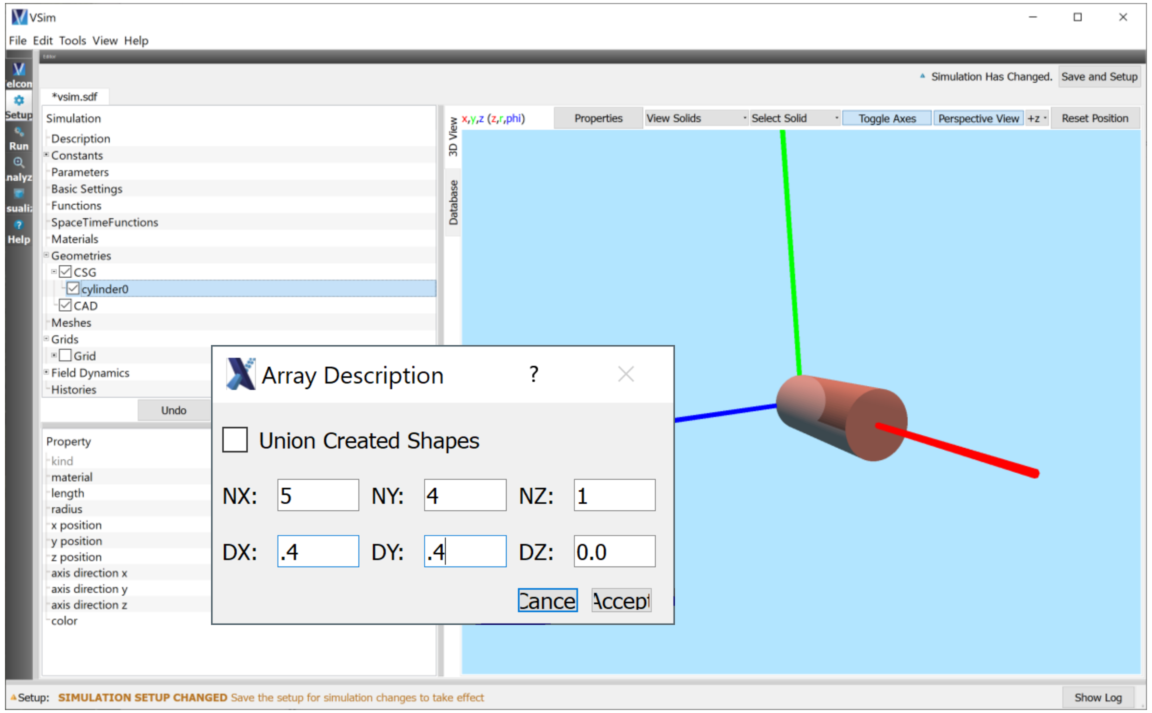


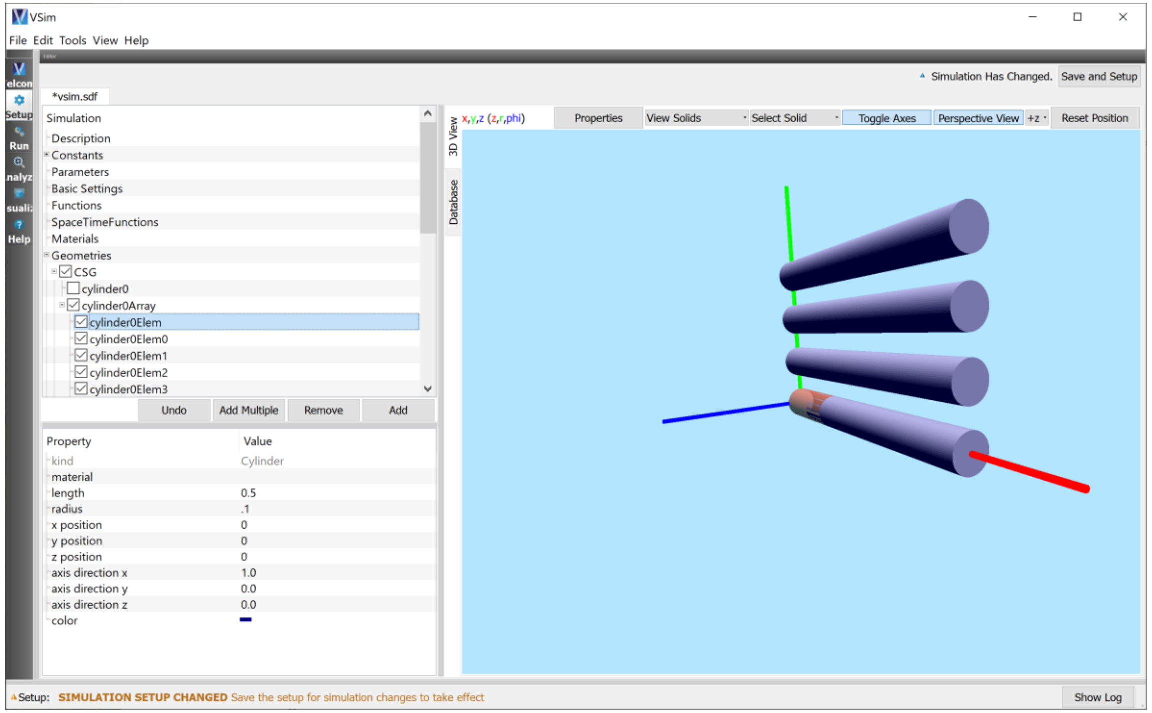
Outline

- Geometry Construction
- Boundary Conditions
- Source
- Far Field Box
- Analysis
- Output

Geometry Construction

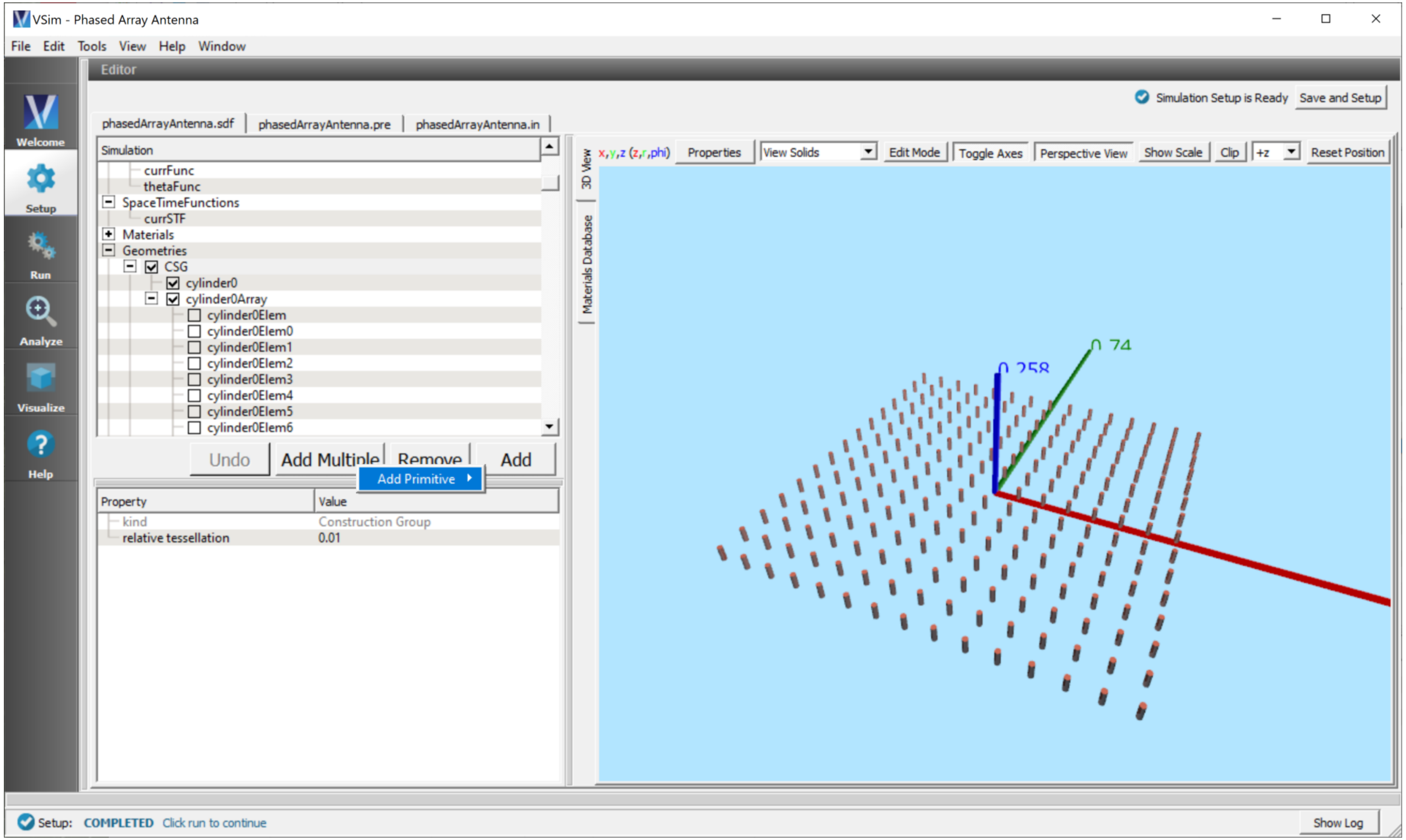
- Uses the “Create Array” option on an existing primitive shape to duplicate the shape with regular spacing.





Our Example

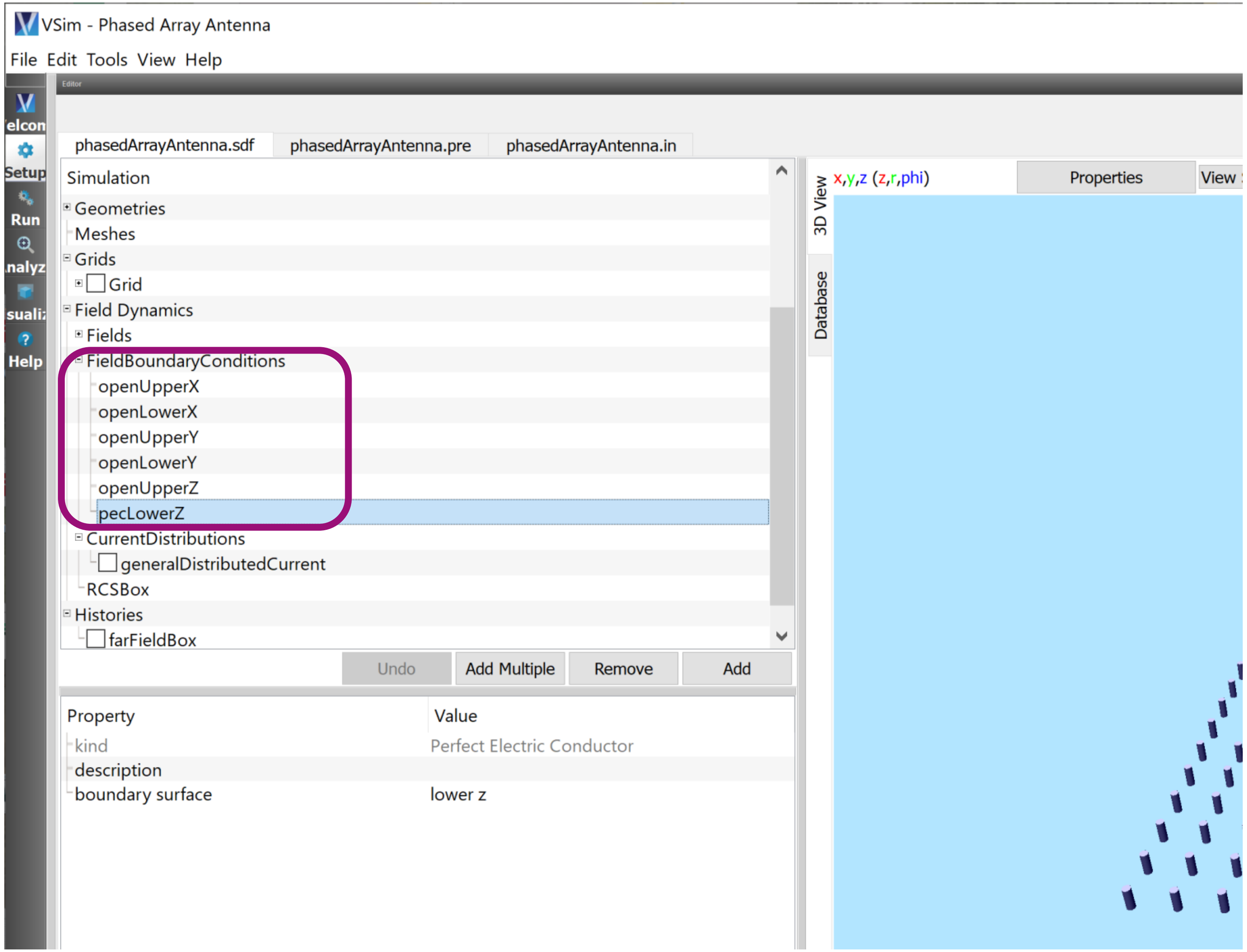
- 15 x 15 array of short cylinders
- Assigned PEC material



Boundary Conditions on 6 Sides

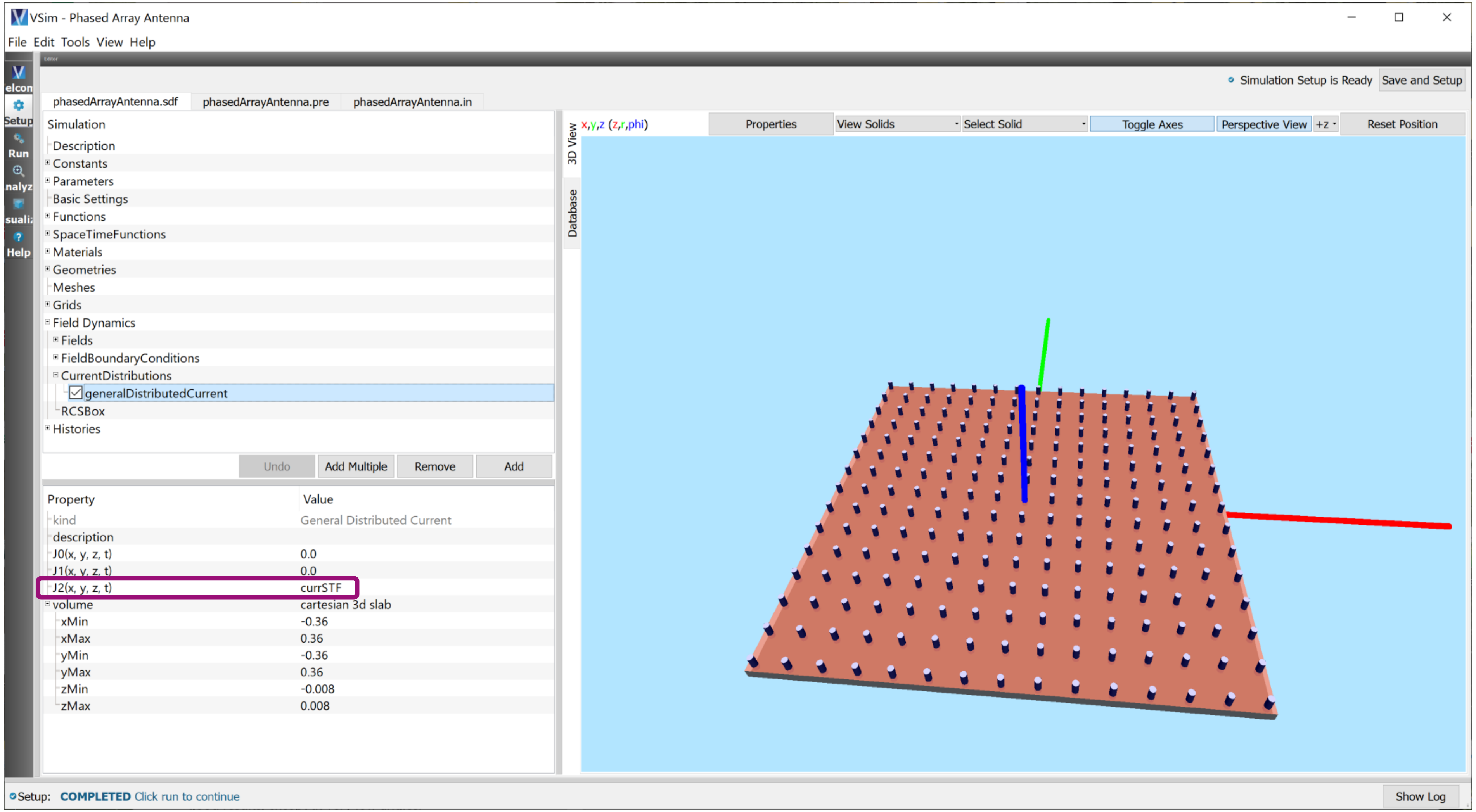
- 5 Open Boundaries
  - Takes up no space.
  - Thick MAL's would be better, if you can afford the mesh.

- 1 PEC Boundary
  - Infinite Ground Plane
  - (does not display)



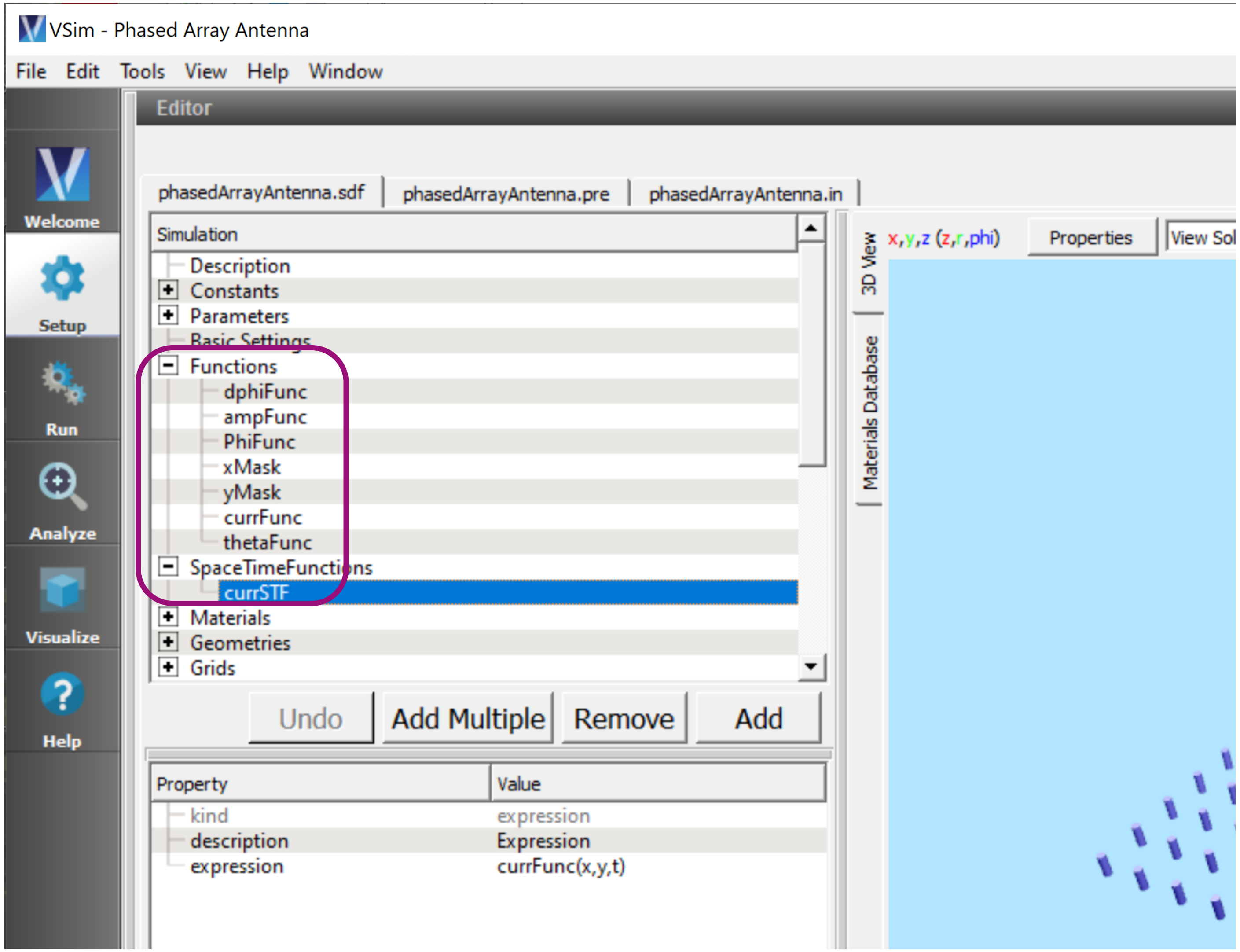
Source drives current from ground plane to radiating elements

- Single source, the entire slab volume between the ground plane and the elements
- Will use a functional "mask" to restrict current to each cylinder
- Jz = currSTF(x,y,t)



Hardest Part of This Example: All The Functions

- dphiFunc
- ampFunc
- PhiFunc
- xMask
- yMask
- currFunc
- thetaFunc
- currSTF



Functions at First Glance: A Lot to Digest

```
currSTF = currFunc(x,y,t)
currFunc(x,y,t) = ampFunc(x,y,t) * sin(PhiFunc(x,y,t)) * xMask(x) * yMask(y)

xMask(x) = H(sin(TWOPI*x/SPACING) - 0.9)
yMask(y) = H(sin(TWOPI*y/SPACING) - 0.9)

dphiFunc(t) = PI/4
thetaFunc(t) = TWOPI*t/T_OFF

PhiFunc(x,y,t) = OMEGA*t + (OMEGA/LIGHTSPEED) * sin(dphiFunc(t)) *
                        (x*cos(thetaFunc(t)) + y*sin(thetaFunc(t)))

ampFunc(x,y,t) = AMP_GAUSS*exp(  ((-1.)/(2*SIGSQR)) *
        ( sin(dphiFunc(t)))^2 * ( x*sin(thetaFunc(t)) - y*cos(thetaFunc(t)) )^2
                        + cos(dphiFunc(t))^2 * (x^2 + y^2)   )
```

Functions Part 1: Plane Wave Phasing

- currFunc(x,y,t) = ampFunc(x,y,t) * sin(PhiFunc(x,y,t)) * xMask(x) * yMask(y)

- This is just: Amplitude * sin(ωt+**k**.**r**) * Mask

- This is the basic starting point for any phased array, the elements are simply driven with phase of a plane wave, having vector wavenumber, **k**.

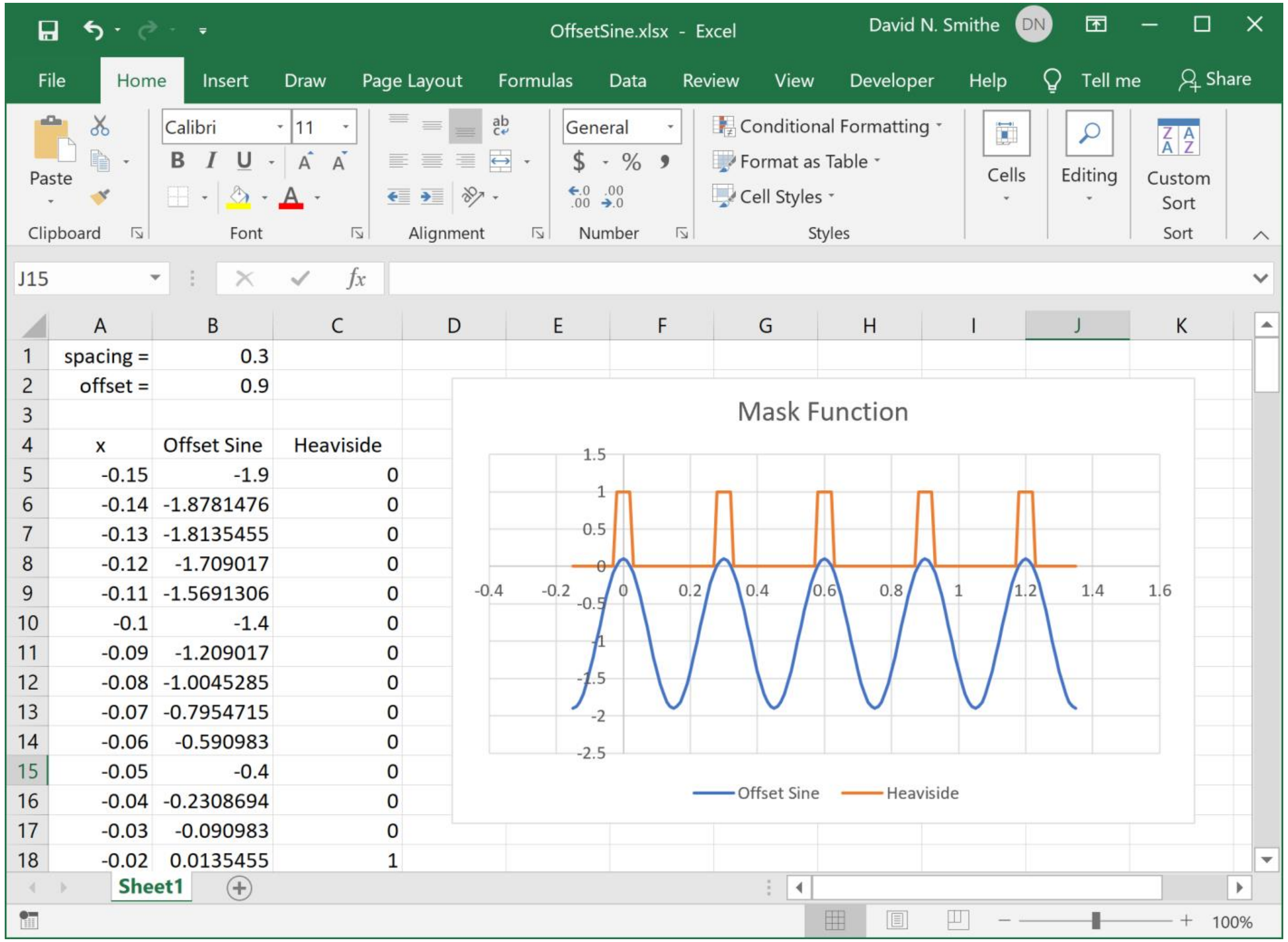
- More advanced corrections can help with side lobes.

Functions Part 2: Masking to the elements

- Heaviside function of offset sine
  - xMask(x) = H(sin(TWOPI*x/SPACING) - 0.9)
  - yMask(y) = H(sin(TWOPI*y/SPACING) - 0.9)

- Product of xMask and yMask gives a thin rectangular source below each cylindrical element.

- Must be smaller than radius, and run from below ground plane into the PEC, in order to prevent charge build up.



Functions Part 3: Plane Wave Phase

- Wavenumber, **k**, is described by the elevation angle, ϕ, and the azimuthal angle, θ.
- In this simulation, $\theta(t)$, and ϕ fixed, e.g., ***beam rotates in time.***

$\mathbf{k} = (\omega/c)\{\mathbf{e}_x \cos\theta \sin\phi + \mathbf{e}_y \sin\theta \sin\phi + \mathbf{e}_z \cos\phi\}$

ϕ: dphiFunc(t) = PI/4

θ: thetaFunc(t) = TWOPI*t/T_OFF

$\omega\, t + \mathbf{k}\cdot\mathbf{x}$

PhiFunc(x,y,t) = OMEGA*t + (OMEGA/LIGHTSPEED) * sin(dphiFunc(t)) *
(x*cos(thetaFunc(t)) + y*sin(thetaFunc(t)))

Functions Part 4: Amplitude

- 1D Gaussian perpendicular to **k** … times 2D Gaussian on array plane.

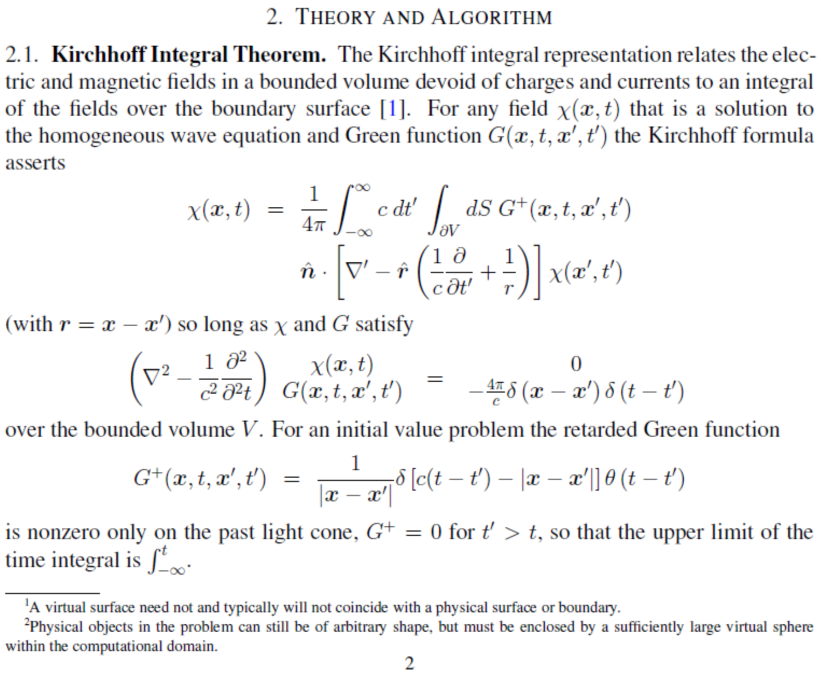
(for low angles) *(for high angles)*

Amplitude: $A \exp(- \{ |(\mathbf{e}_z \times \mathbf{e}_\mathbf{k}) \cdot \mathbf{r}|^2 + |(\mathbf{e}_\mathbf{k} \cdot \mathbf{e}_z)\mathbf{r}|^2 \} / \sigma^2)$

```
ampFunc(x,y,t) = AMP_GAUSS*exp(  ((-1.)/(2*SIGSQR)) *
 ( sin(dphiFunc(t)))^2 * ( x*sin(thetaFunc(t)) - y*cos(thetaFunc(t)) )^2
                                         + cos(dphiFunc(t))^2 * (x^2 + y^2)   )
```

Far Field Box History

- Box surrounds the radiating elements.
- Used in conjunction with Analyzer Tab
  - ComputeFarFieldFromKirchhoffBox
- Kirchhoff Theorem



2. Theory and Algorithm

2.1. **Kirchhoff Integral Theorem.** The Kirchhoff integral representation relates the electric and magnetic fields in a bounded volume devoid of charges and currents to an integral of the fields over the boundary surface [1]. For any field $\chi(\boldsymbol{x},t)$ that is a solution to the homogeneous wave equation and Green function $G(\boldsymbol{x},t,\boldsymbol{x}',t')$ the Kirchhoff formula asserts

$$\begin{aligned}\chi(\boldsymbol{x},t) &= \frac{1}{4\pi}\int_{-\infty}^{\infty} c\,dt' \int_{\partial V} dS\; G^{+}(\boldsymbol{x},t,\boldsymbol{x}',t') \\ &\quad \hat{\boldsymbol{n}}\cdot\left[\nabla' - \hat{\boldsymbol{r}}\left(\frac{1}{c}\frac{\partial}{\partial t'} + \frac{1}{r}\right)\right]\chi(\boldsymbol{x}',t')\end{aligned}$$

(with $\boldsymbol{r} = \boldsymbol{x} - \boldsymbol{x}'$) so long as χ and G satisfy

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial^2 t}\right)\begin{matrix}\chi(\boldsymbol{x},t) \\ G(\boldsymbol{x},t,\boldsymbol{x}',t')\end{matrix} = \begin{matrix}0 \\ -\frac{4\pi}{c}\delta(\boldsymbol{x}-\boldsymbol{x}')\,\delta(t-t')\end{matrix}$$

over the bounded volume V. For an initial value problem the retarded Green function

$$G^{+}(\boldsymbol{x},t,\boldsymbol{x}',t') = \frac{1}{|\boldsymbol{x}-\boldsymbol{x}'|}\delta\left[c(t-t') - |\boldsymbol{x}-\boldsymbol{x}'|\right]\theta(t-t')$$

is nonzero only on the past light cone, $G^{+} = 0$ for $t' > t$, so that the upper limit of the time integral is $\int_{-\infty}^{t}$.

[1] A virtual surface need not and typically will not coincide with a physical surface or boundary.
[2] Physical objects in the problem can still be of arbitrary shape, but must be enclosed by a sufficiently large virtual sphere within the computational domain.

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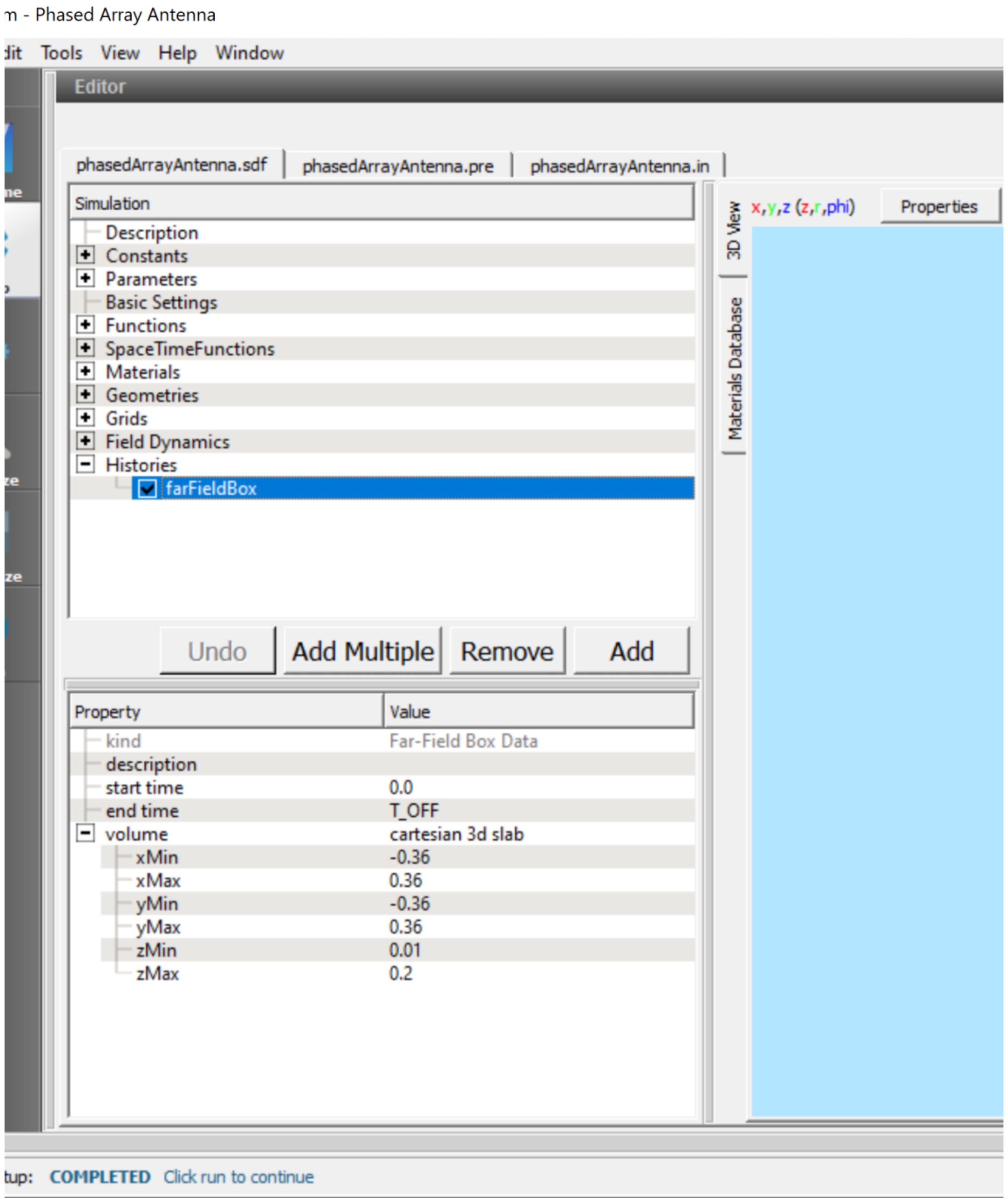
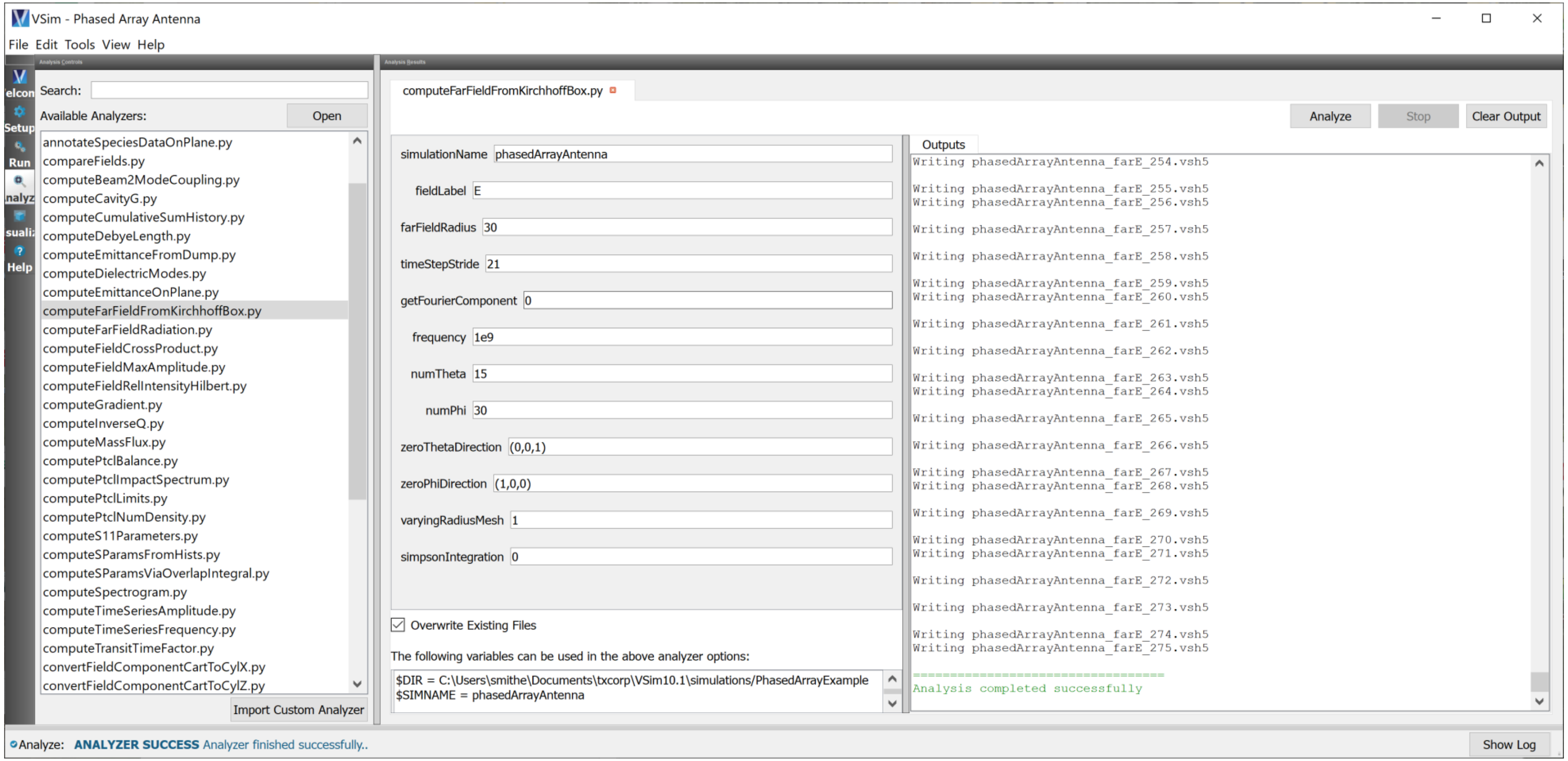


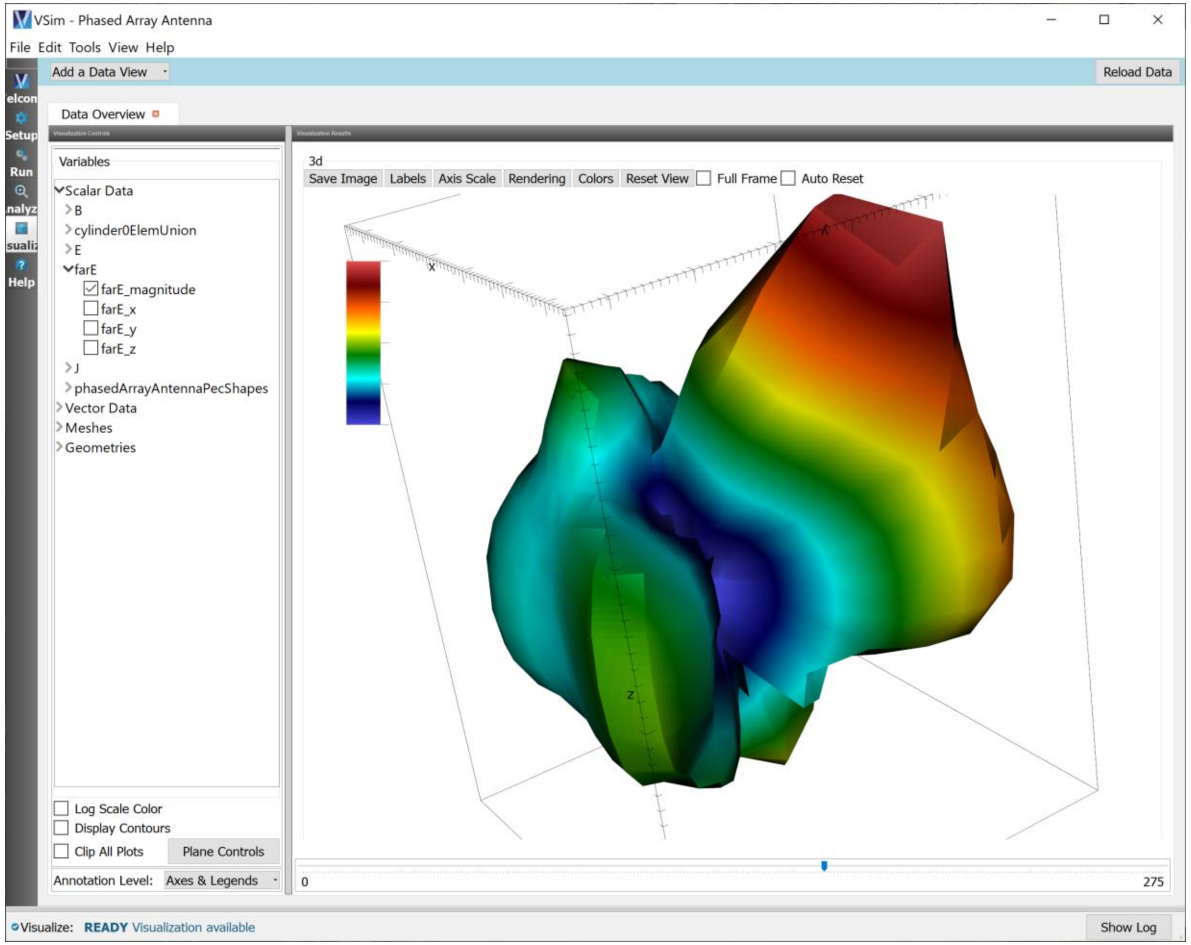
Post Analysis: computeFarFieldFromKirchhoffBox

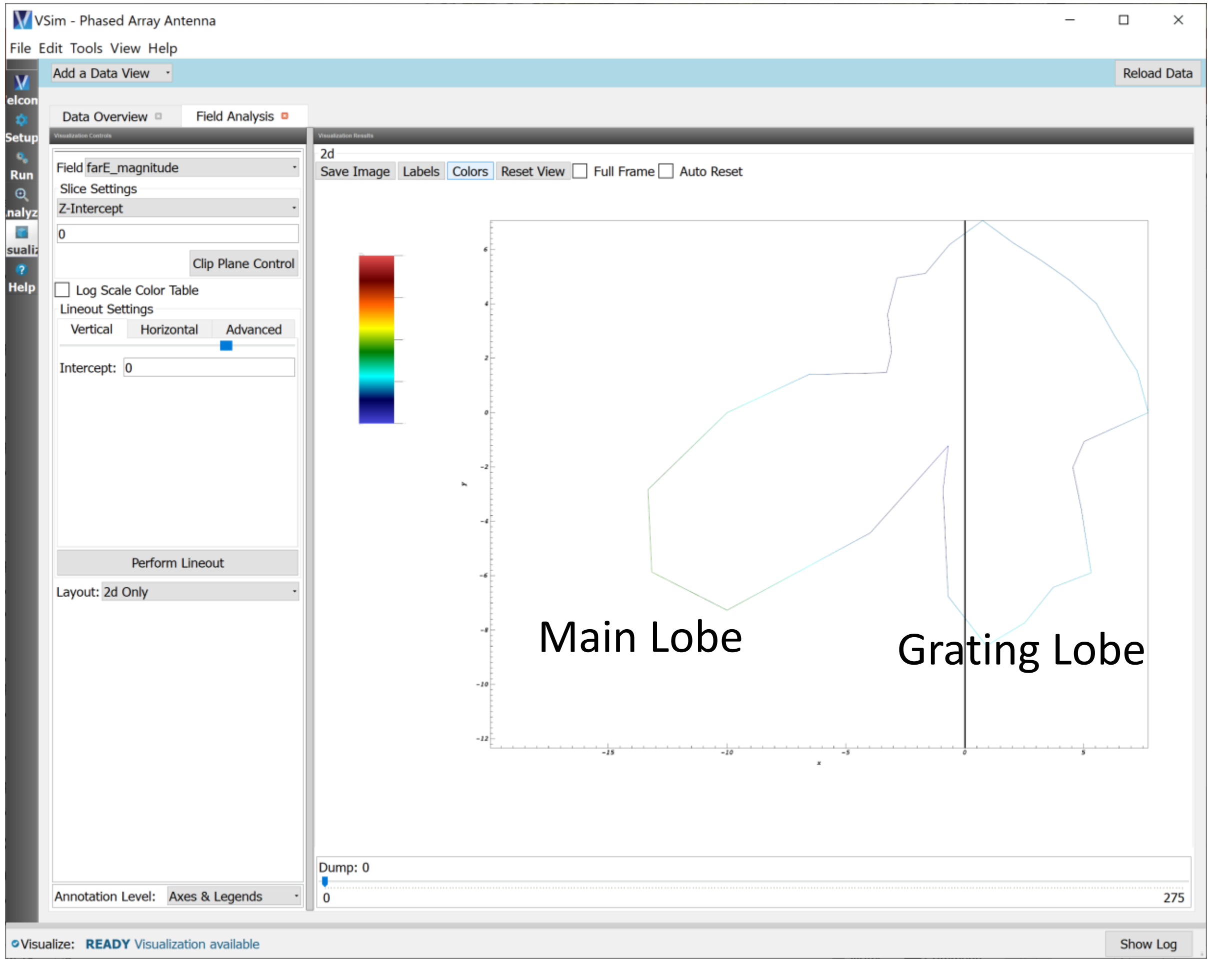
- Pick number of points on “Far Sphere” (numPhi and numTheta)
- Pick number of times on “Far Sphere” (timeStepStride)



Post Analysis Result (farE)

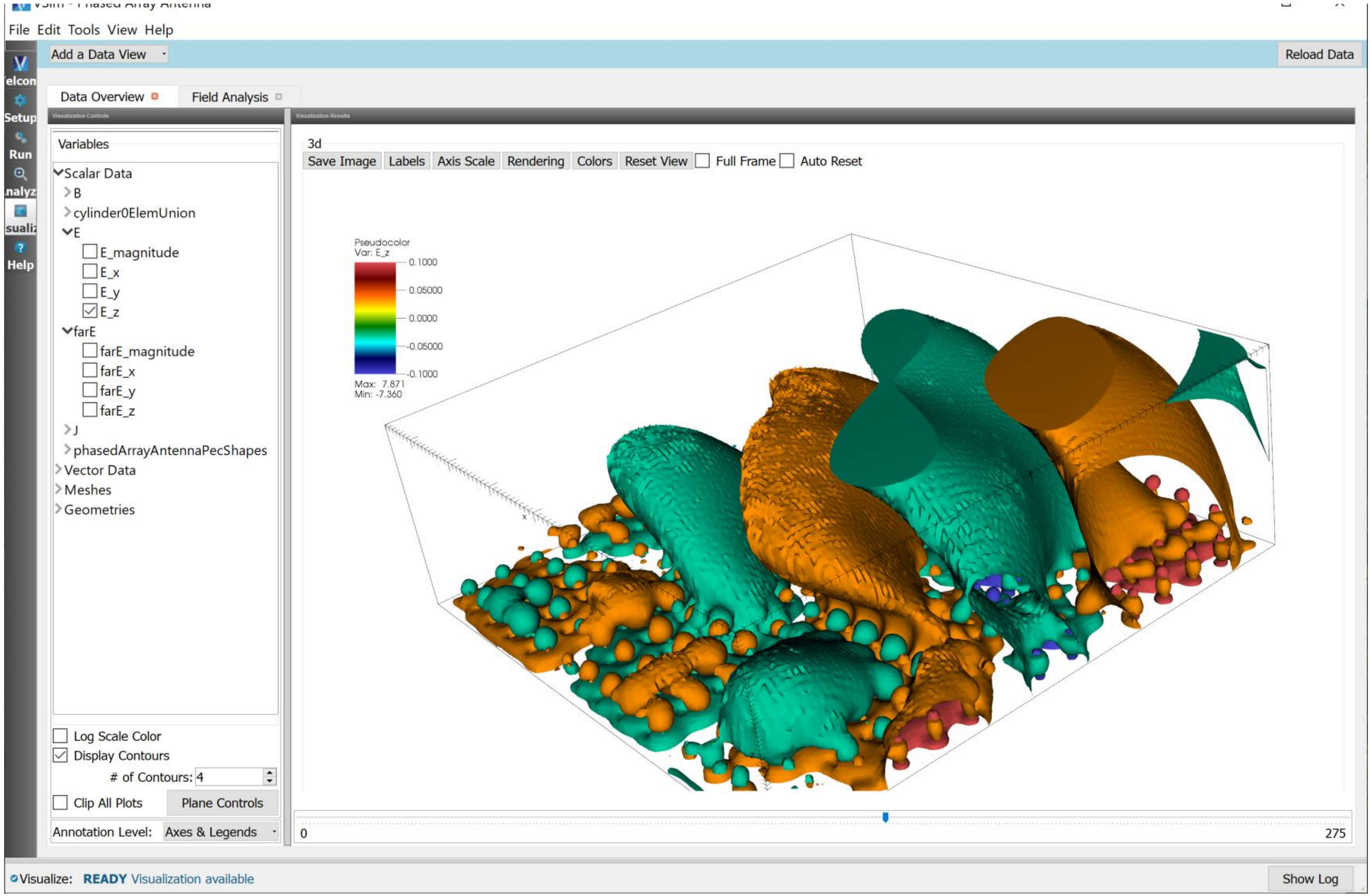
- Instantaneous Far Field Pattern, 3D, and Slice





Visualization of Fields (Ez)

- Display Contours
- Reset Min/Max to +/- 0.1
- Time variation shows beam changing direction



Anticipated Applications

- 5G: Multiple beams / multiple frequencies.
- Optimizing Side Lobes and Grating Lobes.
- Look at cross-talk between nearby arrays on tower.
- Look at near field geometry reflections.
- Look for shadowed regions for indoor installations.

Thank You!

- Questions?